Transportation manuscript No.
(will be inserted by the editor)

A ridesplitting market equilibrium model with utility-based compensation pricing

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Received: date / Accepted: date

Abstract We propose a utility-based compensation pricing method to address the prevalent inequity issue in ridesplitting services. A theoretic equilibrium model is developed at the network level to interpret the intertwined relationships between the endogenous and the decisions in ridesplitting markets. In this paper, the operation property of the market under a common distance-based unified pricing method is described by the response of system performance measures to the decisions in the numerical experiments. Moreover, a gradient descent algorithm is applied to solve the monopoly optimum and social optimum problem under unified pricing. The proposed compensation method is adopted to adjust the individual trip fare determined by the unified pricing based on a predefined compensation function. Specifically, we investigate its effectiveness and influence in the monopoly optimum scenario and social optimum scenario separately. The results show that it can improve the level of services and equity among individual trips without losing any profit and welfare, where the level of services and equity are represented by the mean and the variance of utilities, respectively. Besides, one can even achieve an increment in maximum profit and maximum social welfare under certain conditions. The subsidy schemes at different operation phases of ridesplitting services are also particularly discussed.

Keywords ridesplitting · market equilibrium · pricing · equity · level of services

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This research was supported by the German Research Foundation - Deutsche Forschungsgemeinschaft (DFG) [Project number 392047120, Research grants in collaboration with China].

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1 Introduction

Ridesplitting is an emerging ridesourcing service attracting much attention from industries, governments, individuals, and academics. (i) It has shown invaluable market potential from the day it was launched, which has also been uncovered by theoretical analyses and practices. Transport network companies (TNCs) thus had successively proposed respective ridesplitting programs, such as UberPool, Lyft Line, and Didi ExpressPool (Zhu et al., 2020; Chen et al., 2021; Wang and Yang, 2019; Shaheen and Cohen, 2019; Wang et al., 2021). (ii) Further, its share nature provides the possibility of alleviating the common ailments of modern cities caused by excessive traffic volumes, such as traffic jams and environmental degradation (Zhu et al., 2020; Alonso-Mora et al., 2017; Tachet et al., 2017; Abouelela et al., 2022). Combining people with close itineraries to fill the vacant seats in vehicles is regarded as the most vital method for establishing sustainable transportation systems (Sperling, 2018; Agatz et al., 2012). To unleash the potential for social benefits of ridesplitting, governments are willing to support its development via subsidies or financial aid. (iii) From the perspective of individuals, ridesplitting is preferred due to its superiority in the convenience and flexibility compared to public transport and the affordability compared to hailing a taxi or purchasing a car (Stiglic et al., 2016; Zhu et al., 2020; Li et al., 2021a). (iv) Despite its popularity, the enormous stochasticity and uncertainty in ridesplitting really impede its expansion. Ridesplitting requires more in-depth investigations into operations (e.g., matching mechanism), market assessment (e.g., service placement), and its impacts on multi-modal transportation systems (e.g., modal shift).

Rigorously, ridesplitting is "*a form of ridesourcing where riders with similar origins and destinations are matched to the same ridesourcing driver and vehicle in real time, and the ride and costs are split among users*". This definition was initially presented in Shaheen et al. (2016) and was then adopted in many ridesplitting publications, such as Chen et al. (2017), Li et al. (2019b) and Zheng et al. (2019). To clarify, we want to emphasize that riders with similar travel directions and suitable times (i.e., feasible for all passenger departures and arrivals) can be matched to the same ridesplitting vehicle. The matching procedure is constantly conducted for all vehicles. A ridesplitting vehicle can be dynamically paired with an arbitrary number of requests (fewer than the vehicle capacity) arising within its neighborhood area, regardless of whether it is on the way to pick up a new or drop off an occupant. In other words, ridesplitting allows dynamic matching and route change in real time to combine requests with similar itineraries (Wang and Yang, 2019; Chen et al., 2021; Wang et al., 2019), and vehicles with vacant seats are always in the matching pool.

Mobility providers can optimize the operational objectives by implementing appropriate operating strategies with the help of the market model. While a few market models have been designed for taxi and regular ridesourcing services, such as He et al. (2018) and Ke et al. (2020a), no endeavors have been dedicated to ridesplitting markets. The dynamics in matching and routing complicate the development of ridesplitting market models and thus the comprehensive evaluation of the service. To be more specific, the interdependence among system exogenous (e.g., vehicle fleet size and trip fare) and endogenous variables (e.g., demand and detour time) becomes more complicated due to the uncertain waiting time and detour time caused by the continuity/dynamics in matching and routing. Yet, a thorough understanding of the intertwined relationships between platform decision variables and the system's endogenous variables is essential for the development of optimal operating strategies (Ke et al., 2020a). A reliable market model is also of significant importance for analytically exploring the potential influence of the service.

Further, the competition between ridesplitting and other transport modes imposes more requirements and restrictions on the market model. It is beneficial to construct a market equilibrium (ME) for ridesplitting in the context of multi-modal transportation systems. The ME of ridesplitting is then dependent on the attributes of all available transport modes. Obviously, it will create more challenges in ridesplitting ME modeling due to the modal shift from other transport modes (Zheng et al., 2019). On the other hand, due to the spatial heterogeneity over the market (Alexander and González, 2015), a ME model built at the network level that can reflect the OD demand pattern across the network can offer more accurate insights into ridesplitting markets (Bimpikis et al., 2019). Considering the current percentage of ridesplitting in ridesourcing is relatively low (Tu et al., 2021; Li et al., 2019b; Zheng et al., 2019; Li et al., 2021b), such a ME model can help transportation management agencies and ridesourcing companies tailor the ridesplitting service to adapt to market preference so as to improve the penetration rate (Li et al., 2019b).

Despite the improvement in the level of service (LoS) compared to public transit and taxi services (Ma et al., 2019), the deviation of waiting time and detour time among ridesplitting passengers, together with the conventional distance-based or time-based unified pricing methods, also leads to the inequity of the service. For example, at a platform adopting a distance-based fare structure, the inequity exists between two requests with identical origin and destination but different waiting and detouring because they are charged the same. As passengers are sensitive to the service quality (Wang and Yang, 2019), the inequity problem will discourage the modal shift to ridesplitting services. Hence, to increase the mode share of ridesplitting and thus gain more social benefits, it is imperative to propose appropriate methods to ensure LoS equity. Noteworthy, the pricing method adopted by the service provider is a primary determinant for survivability and sustainability. Specifically, it will influence the attractiveness and competitiveness of the service by acting on user preference, which is mainly affected by monetary cost (trip fare) and time cost (travel time and waiting time) (Qiu et al., 2018; Guan et al., 2019a). Coincidentally, as per de Ruijter et al. (2020) and Ma et al. (2019), trip fare and travel time are also two important indicators of LoS of shared ride services. Therefore, pricing methods are in demand to compensate ridesplitting users based on the initial fee and travel time to enhance the LoS equity of ridesplitting services.

To fill the gaps, this paper first establishes a mathematical model to describe the sophisticated interactions between the decision variables and the system's endogenous variables in the ridesplitting markets with consideration of modal split among multiple transportation modes. The long-term expected values of the endogenous at the stationary equilibrium under a given operation strategy can then be estimated by the ME model. We then derive the optimal operation strategies for profit maximization and social welfare maximization scenarios of ridesplitting services, respectively, for a platform that adopts the distance-based unified pricing method. More importantly, we propose a novel utility-based compensation pricing method to mitigate the inequity among ridesplitting trips and further enhance the LoS.

The remainder of the paper is structured as follows. In Section 2, we review existing studies on ME models and pricing methods in shared ride services. Section 3 formulates a network-based equilibrium model for ridesplitting markets. Section 4 presents the derivation of monopoly optimum and social optimum solutions under distance-based pricing. In Section 5, we propose a utility-based compensation pricing method with the purpose of improving the LoS and equity. In Section 6, numerical experiments are conducted to evaluate the performance of the proposed market model and the effectiveness of the compensation method in different scenarios. Section 7 discusses the limitations of the paper and some future extensions. Finally, conclusions are drawn in Section 8.

2 Related Literature

This section reviews the literature in (i) transportation market equilibrium models that can guide the formulation of ridesplitting markets and representative market scenarios, and (ii) pricing methods in shared ride services that can help understand the principle and consideration behind the fare structure of these services

2.1 Transportation market equilibrium models

Cairns and Liston-Heyes (1996) developed an equilibrium model for the taxi market to understand the competition in the industry. It found that the unregulated industry does not satisfy the conditions of competition, and the existence of equilibrium depends on the regulation of price, entry, and intensity of use of licensed taxis. Besides, it also presented the models of monopoly, the social optimum, and the second-best in the taxi industry. However, it did not consider the spatial difference in demand patterns. An initial attempt to model the taxi market at a network level considering the OD demand pattern was in Yang and Wong (1998). They then improved the model in a series of works by further incorporating demand elasticity and congestion effect (Wong et al., 2001), exploring the impacts of regulatory restraints on the equilibrium (Yang et al., 2002). Furthermore, the improved model was also applied to investigate the performance of nonlinear fare structures on the perceived profitability in Yang et al. (2010). He et al. (2018) gave another shot in modeling the taxi market equilibrium at the network level with specific consideration of both street-hailing and e-hailing modes. The unique reservation-cancellation behavior of e-hailing customers differentiating the two operation modes was explicitly contemplated in the market model.

In some sense, taxi ME models can shed light on the research of equilibrium in the shared transportation markets due to their implicit resemblance. Adopting the modeling framework of previous works on the taxi industry, Ke et al. (2020a) presented an equilibrium model for ridepooling markets and elucidated the complex relationships between endogenous variables and decision variables (trip fare, vehicle fleet size, and allowable detour time). It proved that the monopoly optimum, first-best, and second-best social optimum are always in the regular regime rather than the wild goose chase (WGC) regime[1]. However, it restricted the problem in the situation with at most two passengers sharing a trip. In addition, the market was modeled at an aggregate level without considering the network structure and OD demand patterns.

Contrary to Ke et al. (2020a), where the service provider is the service operator, Bimpikis et al. (2019) formulated the equilibrium state for a matching agency. It pointed out that only when the demand pattern[2] across the network is balanced the benefit of applying spatial price discrimination can be observed. Leveraging the spatial pricing method can promote the demand pattern balance. The result of numerical experiments implied that the total profit and consumers' surplus are maximized at the equilibrium under the optimal pricing policy when the demand pattern of the network is balanced.

Although the models developed in the works above perform well, there is still room for improvement. First, none of them considered passenger preference in the presence of multiple transport modes. The value of time (or willingness to pay) of passengers is the only factor being considered in their modeling framework regardless of the service attributes of other transport options and the relatively constant total travel demand within the city in the short term. This will result in an inaccurate demand estimation when the service attributes are not superior to the others. Second, no work has established a network-based equilibrium model for ridesplitting markets that can capture the travel demand patterns and network characteristics.

2.2 Pricing methods in shared ride services

Demand estimation is the main focus of pricing strategies for shared ride services. Some aim to capture the temporal elasticity of demand to provide optimal solutions for a specific objective (e.g., profit maximization) (Sayarshad and Chow, 2015; Qian and Ukkusuri, 2017). Some try to improve the reliability of the proposed solution by considering the spatial heterogeneity of demand over the network (Chen and Kockelman, 2016; Guo et al., 2017; Qiu et al., 2018; Bimpikis et al., 2019). Furthermore, the users' heterogeneity, which is represented by passenger preference/behavioral models, is also crucial in demand modeling and has been heavily researched in the literature (Chen and Kockelman, 2016; Qiu et al., 2018; Guan et al., 2019a).

Sayarshad and Chow (2015) proposed a non-myopic pricing method for the non-myopic dynamic dial-a-ride problem to maximize social welfare under the assumption of elastic demand. It pointed out that ignoring the elasticity of demand can overestimate the improvement in LoS with non-myopic considerations. Inspired by the demand elasticity among a day, Qian and Ukkusuri (2017) developed a time-of-day

[1] The wild goose chase regime is an inefficient equilibrium where vehicles take substantial time to pick up riders.

[2] Demand pattern of a network is defined as a combination of a demand vector for zones and a weighted adjacency matrix. And it is said to be balanced if, at each zone, the potential demand for rides weakly exceeds the available drivers in the same zone after completing rides.

pricing scheme to maximize the profit for taxi service, where different price multipliers are used to alter trip cost dynamically. It suggested that a strict pricing scheme should consider both temporal heterogeneity and spatial heterogeneity in demand, supply and traffic conditions, together with additional consideration of users heterogeneity in price elasticity. The reservation cancellation behavior at e-hailing taxi platforms could significantly influence the supply management of taxi service and increase the total vacant taxi hours, thus hampering the development of taxi-hailing applications. As such, He et al. (2018) presented a penalty/compensation strategy to restrain such behaviors and further designed optimal strategies to maximize profit for private operators and social welfare for public transportation agencies by a penalty successive linear programming algorithm.

The Multinomial Logit (MNL) model was applied to estimate the mode share of shared autonomous electric vehicle (SAEV) in an agent-based framework in Chen and Kockelman (2016). It investigated the trade-offs between the revenue and mode share of SAEV under different pricing schemes, including distance-based pricing, origin-based pricing, destination-based pricing, and combination pricing strategy. Guo et al. (2017) provided an elaborated demand analysis and dynamic pricing analysis of the ride-on-demand service provided by Shenzhou Ucar in Beijing, China. They adjusted the trip price dynamically by applying appropriate pricing multipliers for different regions based on the demand characteristics in both spatial and temporal dimensions. Integrating the passenger preference, demand distribution, and traffic information of the network, Qiu et al. (2018) proposed a dynamic programming framework to solve the profit maximization problem for a monopolistic private shared mobility-on-demand service (SMoDS) operator. Also, the MNL model was used to model the passenger preference and was integrated into the price optimization model at the request level.

With the consideration of demand difference over the network, Bimpikis et al. (2019) established an infinite-horizon, discrete-time model for ride-sharing services, and explored the impact of the demand pattern on the platform's prices, profits, and the induced consumer surplus. Furthermore, considering the uncertainty of travel time and waiting time in SMoDSs, Guan et al. (2019a) applied the Cumulative Prospect Theory (CPT) to capture the subjective decision making of passengers under uncertainty. A dynamic pricing strategy was proposed on the passenger behavioral model based on CPT, which incorporates a dynamic routing algorithm proposed in Guan et al. (2019b) and thus can provide a complete solution to SMoDSs.

Although the inequity among individual trips is a common phenomenon in the ridesplitting services, it has not been investigated in the existing literature. Thus, it is desirable to have a practical compensation method on the basis of the conventional pricing method (e.g., distance-based unified pricing) to compensate the trips individually according to the respective perceived LoS (e.g., utilities) so as to promote the equity of ridesplitting services.

3 Ridesplitting market equilibrium model

This section develops a network-level ridesplitting market equilibrium model in the context of multi-modal transportation systems. To start at C1 and then, Fig. 1 provides an operational example of a ridesplitting vehicle: the vehicle is navigated to C2 to pick up or drop off a passenger, and a new request arising in C3 is assigned to the vehicle at the same time. The vehicle then has two possibilities: (i) follows the prearranged route to C2 first and then drives to C3 immediately (directly connected) or after accomplishing some missions (indirectly connected, e.g., after driving to C4 to pick up or drop off another passenger), or (ii) is rerouted to pick up the request in C3 first and then drives to C2 immediately or after visiting some places (e.g., C4 for another pick up or drop off, as the mission order may be altered due to the route change). This simple example demonstrates the extreme dynamics of ridesplitting services. We want to emphasize that the picking up status specified for ridepooling vehicles does not exist in this ridesplitting case, as vehicles are always in the matching pool even when they are on the way to pick up passengers. As a result, the pick-up time will be accounted for as a part of passenger waiting time in the ridesplitting market and will not be specifically modeled. Further, due to the presence of other transportation options, passengers who are not matched to ridesplitting vehicles will be naturally counted in the demand for other transport modes. In other words, the probability of using ridesplitting is somewhat representing matching probability, which is consistent with the assumption adopted in Alexander and González (2015). The matching probability in ridesplitting is thus not defined in modeling.

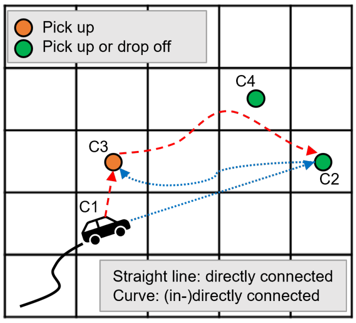


Fig. 1 Operation of ridesplitting serivces.

3.1 System variables in the ridesplitting market

Fig. 2 depicts the interaction between the variables in a ridesplitting market. Variables are categorized into two groups, i.e., exogenous variables and endogenous variables. Exogenous variables include the decision variables of the ridesplitting service and the attributes of other transport options, while endogenous variables are those decided by the ridesplitting market per se, including the waiting time, detour time, ridesplitting demand, and vacant seat hours, etc. The system mainly contains two function modules: pricing method and passenger preference. The adopted pricing method decides the trip fare of ridesplitting services. Passenger preference simulates the interplay between the passenger demand, the expected detour time, and waiting time. The attributes of all transport modes, such as the expected waiting time, expected travel time, and trip fare of public transport, are inputted into the passenger preference module to estimate mode share. Note, with the attributes of other transportation modes fixed, the trip fare and vehicle fleet size of the ridesplitting service are two unique decisions of the system.

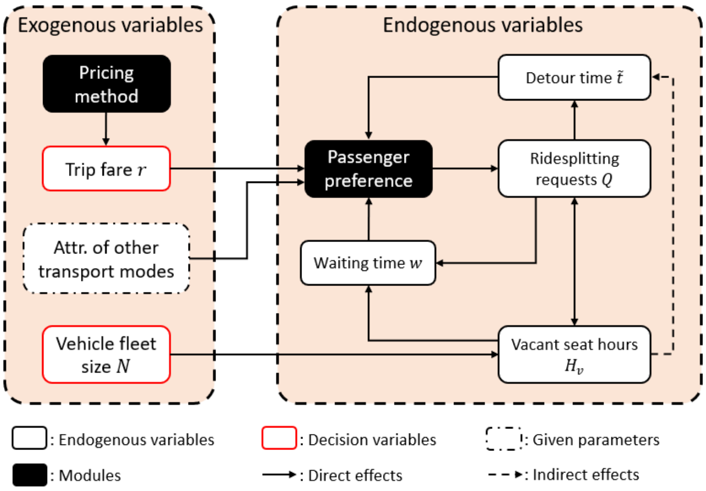


Fig. 2 Relationships between exogenous variables and endogenous variables in the ridesplitting market, adapted from Ke et al. (2020a).

Generally, the expected waiting time is deemed to be related to the number of available vehicles (Cairns and Liston-Heyes, 1996; Li et al., 2019a; Ke et al., 2020a). Considering the sharing nature of the ridesplitting service (ride requests can be matched en route with vehicles that have vacant seats), we revise this assumption as: the expected waiting time depends on the number of available seats. Seat availability is affected by both the vehicle fleet size and ridesplitting demand. Moreover, due to the interdependence among the system endogenous, seat availability also indirectly affects the expected detour time.

In the remainder of this section, how the variables interact is explained in detail, and the method to calculate the ME is presented. Given the exogenous variables, the values of system endogenous variables will then be inherently derived by the ME model.

3.2 Supply of ridesplitting services

Each trip has its origin and destination. Different from the existing works based on abstract and aggregate demand-supply models, we model the ridesplitting market at the network level with consideration of the network structure and traveler OD demand pattern. Consider a network that allows the travel between OD pairs in set $\mathbb{Z}$. For an OD pair i in $\mathbb{Z}$, its origin and destination are denoted as i^o and i^d, respectively. For a given hour (the studying interval is set to one hour), the travel demand for i (i.e., the number of trips from i^o to i^d) is D_i. We denote $P_{i,rs}$ as the mode share of ridesplitting services of i at the ME state, where rs indicates ridesplitting. Then, the passenger demand for ridesplitting from i^o to i^d is estimated by

$$Q_i = D_i P_{i,rs} \tag{1}$$

We know that each seat in ridesplitting vehicles can be either vacant or occupied. We define available seat capacity H_v as the number of vacant seats in stationary equilibrium while utilized seat capacity H_c as the number of occupied seats. For a given hour, the conservation equation of seat capacity is thus given by

$$Nn_s = H_v + H_c \tag{2}$$

where N is vehicle fleet size, n_s is the number of seats in a vehicle.

Note that the travel time for a passenger in ridesplitting services consists of two components: direct trip time (equals to the travel time of driving private cars without detouring) and detour time (due to the detouring to pick-up and/or drop-off other passengers). Thus, the expected travel time of i is given by

$$t_i = t_i^d + \tilde{t}_i \tag{3}$$

where t_i is the expected travel time from i^o to i^d, t_i^d and $\tilde{t}_i$ denote the direct trip time and the expected detour time, respectively. The utilized seat capacity in one hour then can be calculated as

$$H_c = \sum_{i \in \mathbb{Z}} Q_i t_i \tag{4}$$

Substitute Equation (4) into Equation (2) resulting in

$$Nn_s = H_v + \sum_{i \in \mathbb{Z}} Q_i t_i \tag{5}$$

This seat capacity conservation equation bridges the demand for and supply of ridesplitting services and must be satisfied in the ME state. Remarkably, it tells that the total quantity of ridesplitting service supplied to the passengers (Nn_s) is greater than the equilibrium quantity demanded (H_c) by a certain amount of slack (H_v), which is analogous to other mobility service markets. This is also the principle behind the waiting time estimation.

3.3 Demand for ridesplitting services

Fig. 2 shows that the passenger preference model is the core of the market model of the ridesplitting service. Agatz et al. (2012) and Chen et al. (2017) also pointed out that understanding participants' behaviors and preferences is essential for dynamic ridesharing system design and demand modeling, though preferences modeling may be difficult and time-consuming. The preference model needs to aggregate the effects of changes in supply and demand to simulate the response of travelers to the changes. To estimate the ridesplitting passenger demand, we assume all travelers make decisions objectively based on the perceived utilities of the available transport modes. As the easiest and the most used discrete choice model for estimating the travel behaviors of individuals (Train, 2009), the Multinomial Logit (MNL) model has been applied in many aspects of the transportation community, including the transport mode choice behavior (Vrtic et al., 2010; Chen et al., 2013; Krueger et al., 2016). This study also applies MNL to capture passenger preference in the multi-modal transport context. Based on the random utility theory, the utility of choice can be calculated by

$$U = V + \varepsilon \tag{6}$$

where U is the utility, V is the deterministic component of the utility, and ε is the disturbance.

Many factors are closely related to the consumers' intention to use ridesplitting services (Wang et al., 2020), such as personal inventiveness and environmental awareness. We refer to a recent study, Abouelela et al. (2022), a comprehensive investigation of the factors influencing the shift to shared ride services. Nevertheless, time cost and monetary cost are the main factors influencing passengers' choice among the available transportation options. Considering the discrepancy between the perceptions of travel time and waiting time, the utility function (the deterministic part of the utility) of taking one transport mode can be evaluated as

$$V = \beta_t t + \beta_w w + \beta_r r \tag{7}$$

where t, w, and r denote travel time, waiting time, and trip fare. β_t, β_w and β_r are the corresponding preference coefficients.

Assuming the disturbance term ε follows the Gumbel distribution, the probability of one (from i^o to i^d) choosing ridesplitting services is then given by

$$P_{i,rs} = \frac{e^{V_{i,rs}}}{\sum_{j \in \mathbb{M}} e^{V_{i,j}}} \tag{8}$$

where $\mathbb{M}$ is the set of available transport modes in the system, such as private vehicles, public transport and ridesplitting.

Combining Equation (8) and Equation (1), we can estimate the ridesplitting demand for i by (omit the subscript rs in the following text)

$$Q_i = \frac{D_i e^{V_i}}{e^{V_i} + \mu_i} \tag{9}$$

where $\mu_i = \sum_{j \in \{\mathbb{M}-rs\}} e^{V_{i,j}}$ aggregating the utilities of the options other than ridesplitting for ease of representation.

3.4 Expected detour time modeling

Real operations data shows that the average detour time between two passengers in ridesourcing services is inversely proportional to the demand for the service (Ke et al., 2020a,b). Mathematically, the average detour time between two passengers can be estimated by $\tilde{t}^{(2)} = \tilde{A}/\sum_j Q_j$, where $\tilde{A}$ is a market-specific parameter. $\sum_j$ is for $\sum_{j\in\mathbb{Z}}$ in the following text unless otherwise noted. We follow this average detour time model but relax the two-passengers-most restriction (two ride requests are pooled in the ridepooling services at most), which is also adopted in Wang et al. (2021), to the general case. Intuitively, the increase in the number of requests can shorten the average distance between every two passengers, reducing the average detour time. However, it also increases the possibility of pairing more passengers for a vehicle and, therefore, increases the detour time. The subtle contradictory effects of the demand increase should be incorporated into the detour time model.

Let $\bar{t}^d_Q$ denote the mean of direct trip time of all trips, then $\bar{t}^d_Q = \sum_j Q_j t^d_j / \sum_j Q_j$. If there is no detouring, the maximum number of requests a vehicle can serve in one hour (without deadhead time between consecutive requests) is given by $n^{(t)} = 1/\bar{t}^d_Q$. However, due to the limitation of vehicle fleet size, the number of trips assigned to a vehicle is $n^{(a)} = \sum_j Q_j/N$. As a result, the expected number of passengers in a vehicle is n_a/n_t. The detour time of a vehicle is then given by (the time to pick up the first passenger is also counted)

$$\tilde{t}^{(v)} = \frac{n^{(a)}}{n^{(t)}}\tilde{t}^{(2)}_i \tag{10}$$

Moreover, we follows the assumption in Ke et al. (2020a): the detour time of passengers is a fraction of the detour time of vehicles, i.e., $\tilde{t}^{(p)} = \gamma\tilde{t}^{(v)}$, where $\gamma \in (0,1)$. In the ridesplitting markets, vehicles and passengers are matched en route, and vehicles are allowed for detouring within the neighborhood to pick up new requests. This complicates the problem. Nevertheless, it is plausible to impose the following assumption:

Assumption 1 *Given the network structure, trips with longer direct trip time are more likely to have a detour.*

For a OD pair i, we thus introduce a modificator, $t^d_i/\bar{t}^d$, to capture the network spatial difference in the detouring probability. The detour time of i can then be expressed as

$$\tilde{t}_i = \frac{t^d_i}{\bar{t}^d}\tilde{t}^{(p)} = \frac{t^d_i \gamma\tilde{A}\sum_j Q_j t^d_j}{\bar{t}^d N \sum_j Q_j} \tag{11}$$

For simplicity of presentation, we define $A \triangleq \gamma\tilde{A}$ and $A_i \triangleq A t^d_i/\bar{t}^d$, such that

$$\tilde{t}_i = \frac{A_i \sum_j Q_j t^d_j}{N\sum_j Q_j} \tag{12}$$

The expected detour time model implies the detour time is correlated to the spatial characteristics of the network structure ($A_i, \forall i$), the vehicle fleet size (N) and the spatial distribution of ridesplitting demand ($\sum_j Q_j t^d_j$). Note that ridesplitting encompasses many dynamics due to the allowance of dynamic route variation and en route matching. Ridesplitting vehicles can search a wider area for requests, which significantly challenges the integration of vehicle allocation to the detour time modeling. Nevertheless, the vehicle allocation is specifically considered in the waiting time model described in the following subsection.

3.5 Expected waiting time modeling

As per Li et al. (2019a), if assuming the matching process of riders and vehicles follows the Cobb-Douglas type production function, the expected waiting time can be derived to be inversely proportional to the square root of the number of idle vehicles. Considering the sharing nature of ridesplitting services and the possibility of matching en route, we make the following assumption:

Assumption 2 *The waiting time of the ridesplitting service is inversely proportional to the square root of the available seat capacity.*

In addition, we adapt this assumption for fitting network-level modeling by considering (i) the effect of demand over the supply offered to the respective OD pair, (ii) the spatial difference of vehicle allocation, and (iii) the supply attraction relative to demand in the neighborhood. Mathematically, the expected waiting time for trips from i^o to i^d is estimated by

$$w_i = \frac{BQ_i^{\theta}}{\Omega_i \sqrt{\eta_i (Nn_s - \sum_j Q_j t_j)}} \tag{13}$$

where B is a market-specific parameter.

Q_i^{θ} ($\theta > 0$) is used to incorporate the first consideration, i.e., demand over the supply offered to i, where θ measures the intensity of the influence. More demand would produce a “competition” among passengers for the limited supply, especially when the demand is greater than the supply. This is in common with the waiting time model in the network-level e-hailing taxi market constructed in He et al. (2018).

$\eta_i \in (0,1)$ is a percentage value measuring the vacant seat capacity assigned to i, incorporating the second consideration. Note that $\sum_j \eta_j = 1$. The vacant seats are distributed/allocated over the network in accordance with the spatial characteristics of the OD pair, such as the distance of the origin and destination to the city center (denote by λ_{i^o} and λ_{i^d}, respectively, for i) and the OD distance (denote by d_i). The waiting time for OD pairs near the city center is shorter since more vehicles drive through the city center and thus more supply (Li et al., 2019b). Tu et al. (2021) also found that the distance to city center is one of the key influencing factors of ridesplitting ratio. Likewise, the distance between i^o and i^d also determines if the vehicles are willing to detour to catch these requests. For example, Fig. 3 depicts a homogeneous network with 25 zones. The waiting time of trips from C1 to C2 should be less than from C3 to C4, as λ_{C1} is similar to λ_{C3} but λ_{C2} is smaller than λ_{C4}. The waiting time from C1 to C3 should also be short as C1 is close to C3 despite being far from the city center.

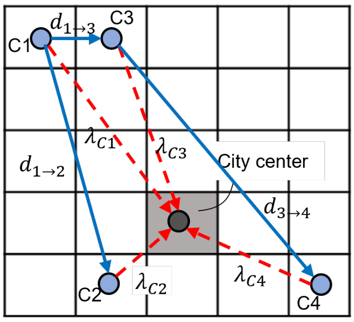


Fig. 3 Consideration of vehicle allocation.

In this study, we apply the form of inverse distance weighting function to calculate the seats distribution as below.

$$\eta_i = \frac{\left((\lambda_{i^o} + \lambda_{i^d})^{\rho_c} d_i^{\rho_d}\right)^{-\rho}}{\sum_j \left((\lambda_{j^o} + \lambda_{j^d})^{\rho_c} d_j^{\rho_d}\right)^{-\rho}} \tag{14}$$

where ρ_c and ρ_d are positive parameters for measuring the influence of the proximity to the city center and OD distance, respectively, and ρ ($\rho > 0$) is the power parameter.

Intrinsically, η_i captures the inherent spatial characteristics of the network structure. In contrast, $\Omega_i(\Omega_i > 0)$ is used to measure the supply attraction caused by the relatively high demand for the neighboring pairs (i.e., the third consideration), which essentially depends on the temporal demand patterns of the market. To clarify, ridesplitting can match passengers with either closer origins or destinations or both (Wang et al., 2019). Specifically, $\Omega_i > 1$ means more vehicles are coming to serve the neighboring pairs of i and vice versa. Here neighboring pairs are defined as follows:

Definition 3.1 Neighboring pairs of i are the OD pairs from the zones within the neighborhood of i^o to the zones within the neighborhood of i^d. The neighborhood of a zone is the area that can reach the zone within a time interval (or a distance threshold) $\bar{\varepsilon}$, including the zone per se.

A graphical example is also provided in Fig. 4. All OD pairs from the neighborhood of i^o to that of i^d are the neighboring pairs of i, such as $C1 \to i^d$. Ω_i, named as supply attraction factor, is given by

$$\Omega_i = \frac{n_z \sum_{j\in\mathbb{Z}_i} Q_j}{\sum_{k\in\mathbb{Z}} \sum_{j\in\mathbb{Z}_k} Q_j} \tag{15}$$

where n_z is the number of OD pairs in $\mathbb{Z}$, $\mathbb{Z}_i$ is the set of neighboring pairs of i which is a subset of $\mathbb{Z}$. Ω_i also reflects that ridesplitting vehicles are allowed deviating from a given path within a service area, which is a commonality with the Mobility Allowance Shuttle Transport (MAST) service (Quadrifoglio et al., 2008).

For simplicity of presentation, we define $B_i \triangleq B/\sqrt{\eta_i}$, such that

$$w_i = \frac{B_i Q_i^{\theta}}{\Omega_i \sqrt{N n_s - \sum_j Q_j t_j}} \tag{16}$$

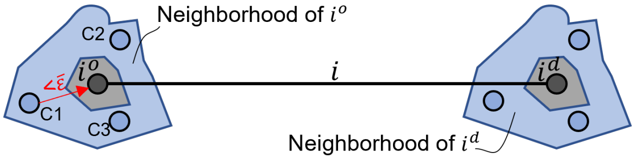


Fig. 4 Definition of buffer areas.

3.6 Equilibrium in the ridesplitting markets

From Fig. 2, we know that both expected detour time $\tilde{t}$ and expected waiting time w are related to the vehicle fleet size N and ridesplitting demand Q (H_v can be replaced by N and Q). Here Q is the vector of ridesplitting demand for all OD pairs. Thus, with a slight abuse of notation, we can rewrite detour time and waiting time as $\tilde{t}(Q,N)$ and $w(Q,N)$, respectively. Recall that the expected travel time is the sum of direct trip time and expected detour time, so we can rewrite the travel time as $t(Q,N)$. The utility function for ridesplitting services is then given by

$$V_i(Q,N) = \beta_t t_i(Q,N) + \beta_w w_i(Q,N) + \beta_r r_i \tag{17}$$

Substituting Equation (17) into Equation (9), the ridesplitting passenger demand thus becomes an implicit function of itself.

$$Q_i = \frac{D_i e^{V_i(Q,N)}}{e^{V_i(Q,N)} + \mu_i} \tag{18}$$

The ME in the ridesplitting market is the ultimate stable state of the market (the supply-demand interaction eventually damps out), at which the relationships between the system endogenous variables (e.g., passenger demand, average detour time) can be satisfied under a specific operation strategy (e.g., vehicle fleet size, trip fare). Mathematically, the demand-supply equilibrium is established when both demand and supply equations are satisfied simultaneously (Arrow and Debreu, 1954). More specifically, under certain operation strategies, an equilibrium in a ridesplitting market is a set of values of $\tilde{t}_i, w_i$ and Q_i that satisfies the equations system composed of

Equation (5), (12) and (16)-(18) for all i in $\mathbb{Z}$. For convenience, we put them below.

$$\begin{aligned}
&Nn_s = H_v + \sum_i Q_i t_i \\
&\tilde{t}_i = \frac{A_i \sum_j Q_j t_j^d}{N \sum_j Q_j}, \forall i \in \mathbb{Z} \\
&w_i = \frac{B_i Q_i^\theta}{\Omega_i \sqrt{Nn_s - \sum_j Q_j t_j}}, \forall i \in \mathbb{Z} \\
&\Omega_i = \frac{n_z \sum_{j \in \mathbb{Z}_i} Q_j}{\sum_{k \in \mathbb{Z}} \sum_{j \in \mathbb{Z}_k} Q_j}, \forall i \in \mathbb{Z} \\
&V_i(Q,N) = \beta_t t_i(Q,N) + \beta_w w_i(Q,N) + \beta_r r_i, \forall i \in \mathbb{Z} \\
&Q_i = \frac{D_i e^{V_i(Q,N)}}{e^{V_i(Q,N)} + \mu_i}, \forall i \in \mathbb{Z}
\end{aligned} \tag{19}$$

It is worth pointing out that Equation (5) and the set of Equation (18) given different i describe the supply of and demand for ridesplitting services, respectively. In practice, this equations system can be solved via a hybrid method for nonlinear equations proposed in Powell (1970). Our numerical experiments indicate that the resultant solutions are always unique under rational operation strategies.

4 Market scenarios and optimal operation strategies under distance-based unified pricing

This study investigates the inequity problem among individual trips in ridesplitting services caused by the combined effect of deviation in travel time and the commonly used unified pricing. Ridesplitting services can be operated by either private companies or public transportation agencies, leading to the difference in operation objective and inequity severity. Hence, it is very important to analyze the market performance under the unified pricing method and compare different market scenarios. This section introduces two representative scenarios extensively discussed in the literature and shown in actual operation as below.

(1) Monopoly scenario. A monopolist aims to maximize its profit.
(2) Social optimum scenario. The platform aims to maximize social welfare.

As mentioned in Li et al. (2019b), the payment of ridesplitting services is mostly based on the actual travel time and distance of the trip. Without loss of generality, based on the proposed network equilibrium, we derive an algorithm to find the optimal solutions for the two scenarios under distance-based unified pricing. One should be able to extend it to the case of time-based unified pricing accordingly. The fare structure used in the distance-based unified pricing method is such that trip fares are proportional to the travel distance. The trip fare of i is given by

$$r_i = pd_i \tag{20}$$

where p is the unit price. The remainder of this section first expounds on the monopoly and social optimum scenarios, and then presents the optimization algorithm for finding out the monopoly optimum (MO) and the social optimum (SO) for the two scenarios. A utility-based compensation pricing method is then proposed in the next section and will be applied to enhance the equity among individuals in the MO and SO operation scenarios separately.

4.1 Monopoly scenario

The ridesplitting service is operated with for-hire drivers and vehicles, which differs it from ridesharing. A ridesplitting monopolist attempts to maximize its profit by optimizing the vehicle fleet size and trip fare. Profit is the difference between revenue and operating cost. The problem can then be formulated as

$$(\mathbf{P1})\ \text{maximize}\ \Pi(N,r) = \textstyle\sum_i D_i P_i r_i - \phi N \tag{21}$$

where ϕ is the operating cost of a vehicle in one hour, r is the vector of trip fare of all OD pairs. The first-order optimality conditions of this problem are:

$$\frac{\partial \Pi}{\partial p} = \sum_i \left(D_i \frac{\partial P_i}{\partial p} r_i + Q_i d_i \right) = 0 \tag{22}$$

$$\frac{\partial \Pi}{\partial N} = \sum_i D_i \frac{\partial P_i}{\partial N} r_i - \phi = 0 \tag{23}$$

After some straightforward work we arrive at:

$$\frac{\partial \Pi}{\partial p} = \sum_i D_i P_i (1-P_i) \left(\beta_t \frac{\partial t_i}{\partial p} + \beta_w \frac{\partial w_i}{\partial p} + \beta_r d_i \right) r_i + \sum_i Q_i d_i = 0 \tag{24}$$

$$\frac{\partial \Pi}{\partial N} = \sum_i D_i P_i (1-P_i) \left(\beta_t \frac{\partial t_i}{\partial N} + \beta_w \frac{\partial w_i}{\partial N} \right) r_i - \phi = 0 \tag{25}$$

Due to the sophisticated interdependence among system endogenous variables (e.g., waiting time, detour time, ridesplitting demand), the first-order conditions of **P1** cannot be solved analytically. Thus, the algorithm presented later in Section 4.3 will be used for this purpose. One may note that the derivatives of profit are functions of the derivatives of detour time and waiting time. For ease of reading, the method for calculating the derivatives of detour time and waiting time with respect to p and N is omitted here and can be found in Appendix B.

4.2 Social optimum scenario

Social welfare also known as social surplus, equals the sum of consumers' and producers' surplus (Cairns and Liston-Heyes, 1996). Mathematically, the social welfare maximization problem can be constructed as

$$(\mathbf{P2})\ \text{maximize}\ S(N,p) = \textstyle\sum_i \int_0^{Q_i} F_i(x)dx - \phi N \tag{26}$$

where $F_i(\cdot)$ is the inverse of the demand function given in Equation (9). From Equation (9), we can easily get

$$F_i(x) = r_i = \frac{1}{\beta_r}\left[\ln x - \ln(D_i - x) + \ln \mu_i - \beta_t t_i - \beta_w w_i\right] \tag{27}$$

Note that the consumers' surplus must be carefully determined due to the inclusion of waiting time and detour time in the demand curve. The integral in **P2** is obtained by integrating under a hypothetical demand curve in which the service level (waiting time, detour time) is held fixed while the trip fare varies, rather than under the real market demand curve (Anderson and Bonsor, 1974; Cairns and Liston-Heyes, 1996).

After some straightforward work we can write the first-order conditions of **P2** as

$$\frac{\partial S}{\partial p} = \sum_i \left[\frac{Q_i}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial p} - \beta_w \frac{\partial w_i}{\partial p}) + D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p} + \beta_w \frac{\partial w_i}{\partial p} + \beta_r d_i) F_i(Q_i)\right] = 0 \tag{28}$$

$$\frac{\partial S}{\partial N} = \sum_i \left[\frac{Q_i}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial N} - \beta_w \frac{\partial w_i}{\partial N}) + D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial N} + \beta_w \frac{\partial w_i}{\partial N}) F_i(Q_i)\right] - \phi = 0 \tag{29}$$

Similarly, the first-order conditions of **P2** cannot be solved analytically. We can see that the derivatives of welfare are also functions of derivatives of detour time and waiting time.

4.3 Gradient Descent algorithm for optimizing operation strategies

Due to the difficulty in tackling the first-order optimality conditions of **P1** and **P2**, we will apply the Gradient Descent (GD) algorithm to approximate the optimal solutions for the problems. Keller (2013) proved that under certain conditions, the local maximum of the non-convex problem **P1** in terms of the prices is also a global maximum. Moreover, the numerical experiments conducted in the existing literature on taxi markets (Yang et al., 2002) and ridepooling markets (Ke et al., 2020a) also found that, in rational ranges of regulated decisions, the local optimums of problems **P1** and **P2** in terms of price and vehicle fleet size are also the global optimums. It means that though gradient-based algorithms only ensure convergence to the local minimum, they can be applied to solve problems **P1** and **P2**. The experiment results in this study show that the proposed algorithm can solve the two problems effectively, and the network equilibrium for each combination of decision values is unique in general.

Define the operation strategy as $O \triangleq (N, p)$. Let $\mathscr{J}(O)$ denote the objective function, which can be either $\Pi(O)$ or $S(O)$. Then the GD for solving the relevant optimization problems can be described as Algorithm 1.

Algorithm 1 Gradient Descent for solving unified pricing optimization problems

1: Initialize the operation strategy $O^{(0)}$, algorithm step size ψ, maximum number of iterations τ_{max}
2: Initialize the iteration index: $\tau = 0$
3: **for** $\tau < \tau_{max}$ **do**
4: Compute the market equilibrium state under $O^{(\tau)}$:
(1) Compute trip fare $r_i, \forall i$ via Equation (20)
(2) Compute ridesplitting demand $Q_i, \forall i$, expected detour time $\tilde{t}_i, \forall i$, expected waiting time $w_i, \forall i$ via Equation (19)
5: Compute the partial derivatives of travel time $t_i, \forall i$ and waiting time $w_i, \forall i$ with respect to unit price p and vehicle fleet size N under $O^{(\tau)}$, respectively:
(1) Compute the partial derivatives with respect to p via system {Equation (48) to (51), $\forall i$}
(2) Compute the partial derivatives with respect to N via system {Equation (52) to (55), $\forall i$}
6: Compute the partial derivatives of the objective function $\frac{\partial \mathscr{J}}{\partial p}, \frac{\partial \mathscr{J}}{\partial N}$ under $O^{(\tau)}$:
(1) Monopoly scenario: compute $\frac{\partial \mathscr{J}}{\partial p}$ via Equation (24); compute $\frac{\partial \mathscr{J}}{\partial N}$ via Equation (25)
(2) Social optimum scenario: compute $\frac{\partial \mathscr{J}}{\partial p}$ via Equation (28); compute $\frac{\partial \mathscr{J}}{\partial N}$ via Equation (29)
7: The gradient direction: $g^{(\tau)} = \nabla \mathscr{J}(O^{(\tau)}) = \left[\frac{\partial \mathscr{J}}{\partial p}, \frac{\partial \mathscr{J}}{\partial N}\right]$
8: **if** $g^{(\tau)}$ does not change ($< \varepsilon_{max}$) **then**
9: Output the optimal solution: $O^* = O^{(\tau)}$
10: **break**
11: **end if**
12: Improve the operation strategy: $O^{(\tau+1)} = O^{(\tau)} + \psi g^{(\tau)}$
13: Update the iteration index: $\tau = \tau + 1$
14: **end for**

5 Utility-based compensation pricing

Though ridesplitting offers a promising increase in LoS compared to public transit and taxi services (Ma et al., 2019), the variance in waiting time and detour time due to the dynamically matching and routing can easily render the inequity of LoS among riders at platforms adopting simple unified pricing methods. Kleiner et al. (2011) and Wang and Yang (2019) pointed out that the variance in the potential detour distances plays a negative role in the popularity of ride-shares. As a result, more efforts are needed to innovate appropriate methods to enhance the LoS equity in ridesplitting services. Considering the tight connection between LoS and the perception of the utility, we propose a utility-based compensation pricing method in this section, giving the first shot on the problem.

Due to the discrepancy of ODs and the uncertainty involved in ridesplitting services, passengers always pay different monetary and time costs for trips. This causes inequity among passengers, represented by the variance of LoS. In Wang et al. (2018), the quality of matching in ridesplitting services is typically defined as the sum of the utilities of all individuals in the system. de Ruijter et al. (2020) and Ma et al. (2019) also suggested that trip fare and travel time are essential indicators of LoS of shared ride services. Inspired by the literature, we introduce the following assumption to quantify the LoS of ridesplitting services.

Assumption 3 *The level of service of a ridesplitting trip can be represented by the corresponding utility.*

Notice that the utility function applied in this study is a linear combination of trip fare, waiting time, and travel time, which captures the main factors affecting passengers' perception; thus, it is plausible to hypothesize the utility is somehow tantamount to LoS. Then, the proposed utility-based compensation pricing method is to improve the equity among passengers by reducing the variance of trips' utilities calculated on the basis of the initial fee (unified pricing), travel time, and waiting time. In particular, by this compensation approach, trips with an initial utility less than a predefined utility threshold will be compensated based on a platform-determined function. Apart from equity, it is also expected to promote the LoS to some extent. Furthermore, an increase in waiting time and detour time could be observed due to the attraction of more ridesplitting demand because of the improvement in LoS and equity.

The remainder of this section first elucidates the compensation principle adopted and then presents the method to approximate the new equilibrium state after applying compensation pricing.

5.1 Compensation principle

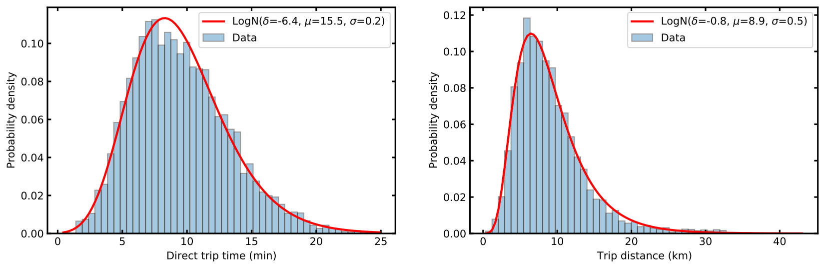
To define the compensation principle, we need to specify the utility threshold value (termed as the compensation reference point, CRP) and the method to calculate the amount of compensation (termed as compensation function). Trips below CRP will be compensated with an amount of money determined by the compensation function. Moreover, in order to connect CRP with observed utilities of trips and thus guarantee the compatibility and viability of the compensation method in markets with different characteristics, we define CRP as a proportion of the mean of trips utilities, which can be written as

$$a = \alpha \bar{V} \tag{30}$$

where α is named as the compensation reference factor (CRF). Intrinsically, the compensation approach influences LoS and equity from two directions: (i) For the trips whose initial utility is less than $\alpha \bar{V}$, their utilities will increase attributed to the compensation; (ii) For the trips whose utility is greater than $\alpha \bar{V}$, however, their utilities will reduce due to the increasing of waiting time and detour time contributed by the induced demand. As a consequence, it is plausible to observe the improvement of equity. Besides, the improvement of LoS in the following experiments demonstrates that the former effect is stronger than the latter.

In the unified pricing method, system endogenous variables under certain operation strategy (N, p) can be computed directly through the ME model. Trips are considered at the network level, and variables are aggregated based on ODs. Differently, the compensation method developed in this section is individual-based. The results in Chen et al. (2017), Li et al. (2019b), Zheng et al. (2019) and Chen et al. (2021) showed that the travel distance, travel time and waiting time follow log-normal distribution. On the other hand, we also can estimate well-fitted log-normal distributions for the direct trip time and trip distance from the simulation data that will be used in our case studies, as shown in Fig. 5. In Fig. 5, δ is the shift from zero, and μ and σ are the mean and standard deviation of the variable's natural logarithm, such that $\ln(X+\delta) \sim \mathscr{N}(\mu,\sigma)$ where variable X is travel distance or travel time. Since travel time is the sum of waiting and detour time, detour time also follows a log-normal distribution. Thus, we make the following assumptions for the related variables to ease processing.

Assumption 4 *Attribute x (i.e., w, d, $\tilde{t}$ and t^d) of trips from i^o to i^d follows a log-normal distribution with δ_x, μ_x and σ_x as the shift from zero, mean and standard deviation, respectively.*



(a) Direct trip time (b) Direct travel distance

Fig. 5 Distribution of simulated direct trip time and travel distance.

The parameters of direct trip time distribution and trip distance distribution can be directly estimated from the simulation data of the respective ODs. We assume there is no shift from zero for the waiting time and detour time distributions. Note, for a variable X following log-normal distribution (no shift), its expected value and variance can be estimated by

$$\mathrm{E}(X) = e^{\mu+\frac{1}{\sigma}} \tag{31}$$

$$\mathrm{Var}(X) = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} \tag{32}$$

from which we can get

$$\mu = \frac{1}{2}\left[4\log\left(\mathrm{E}(X)\right) - \log\left(\mathrm{Var}(X)+\mathrm{E}^2(X)\right)\right] \tag{33}$$

$$\sigma^2 = \log\left(\mathrm{Var}(X)+\mathrm{E}^2(X)\right) - 2\log\left(\mathrm{E}(X)\right) \tag{34}$$

The means of waiting time and detour time are known. After assuming the standard deviations based on the means empirically, we can obtain the parameters of the log-normal distribution generators with the above estimators. The attributes of individual trips can then be generated from the fitted distributions.

With Assumption 4, we can calculate the utility of each individual trip by

$$V_{i,k} = \beta_t t_{i,k} + \beta_w w_{i,k} + \beta_r r_{i,k} \tag{35}$$

where k is the index of the trip, and $t_{i,k} = \tilde{t}_{i,k} + t^d_{i,k}$. After compensation, the trip fare is given by

$$r^a_{i,k} = pd_{i,k} + c_{i,k} \tag{36}$$

where $c_{i,k}$ is the amount of compensation, $r^a_{i,k}$ is the trip fare after compensation. Recall that the utility of a trip after compensation must satisfy a predefined function termed as compensated utility function, which describes the relationship between the utility before and after compensation. The difference between the initial utility and the utility resulted from the compensated utility function determines the amount of compensation. Specifically, the compensation is given by

$$c_{i,k}(V_{i,k}) = \frac{V^a_{i,k} - V_{i,k}}{\beta_r} \tag{37}$$

$$V^a_{i,k} = f^a(V_{i,k}) \tag{38}$$

where $V_{i,k}$ is the utilities before compensation, $V^a_{i,k}$ is the target utility, and $f^a(V_{i,k})$ is the compensated utility function for calculating the target utility, which is a function of $V_{i,k}$. Rigorously, We define the compensated utility function $f^a(V_{i,k})$ as follow.

Definition 5.1 A compensated utility function is a function that describes the relationship between the utilities of trips before and after compensation. The compensated utility function should hold the following properties.

(1) The utility after compensation should not be larger than the compensation reference point, i.e., $V^a_{i,k} \leq a, \forall i,k$.
(2) The order of trips sorted by utility should not change after compensation, i.e., if $V_{i_1,k_1} \leq V_{i_2,k_2}$, then $V^a_{i_1,k_1} \leq V^a_{i_2,k_2}, \forall i_1, i_2, k_1, k_2$.
(3) Trips with a utility farther below the compensation reference point should get more compensation than those closer, i.e., if $V_{i_1,k_1} \leq V_{i_2,k_2} \leq a$, then $c_{i_1,k_1} \geq c_{i_2,k_2} \geq 0, \forall i_1, i_2, k_1, k_2$.

5.2 Market equilibrium

Due to the changes in the LoS of ridesplitting services, a new balance will present. For the purpose of comparative analysis, we need to estimate the market equilibrium for the case under the proposed compensation pricing method. To this end, trips need to be re-aggregated according to the ODs to estimate the new equilibrium. The new OD-based trip fare is given by

$$\check{r}_i = \frac{1}{D_i} \sum_k r^a_{i,k} \tag{39}$$

The equilibrium under compensation pricing will then be explained by the equations system provided in Equation (19) after plugging the new trip fare. It is worth pointing out that, in practice, the CRP for a market is decided based on the historical operation data.

Table 1 Estimation of preference coefficients.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.589	0.0509	-11.6	0
β_t (/min)	-0.128	0.0139	-9.18	0
β_w (/min)	-0.113	0.0134	-8.46	0

6 Numerical experiments

In this section, we introduce the study area, data, and parameters used for the experiments first. Next, we evaluate the performance of the proposed ridesplitting market on the operational objectives described in Section 4, i.e., profit maximization and social welfare maximization, and evaluate the influence of decision variables on the system variables and objectives. We then analyze the difference in the market under the proposed utility-based compensation pricing method and further explore its impact and effectiveness on the LoS and service equity.

6.1 Experiment setups

Tsiamasiotis et al. (2021) designed and performed a web-based stated-preference survey to identify factors affecting the travel behavior of passengers due to the introduction of dynamic vanpooling services. 27 hypothetical scenarios were created and divided into three blocks. In each scenario, three alternatives were provided, including private car, public transport, and dynamic vanpooling. Respondents were asked to state their preference in a five-point rating scale given the values of in-vehicle travel time, monetary cost, and waiting time (including walking time), which conforms to the requirement of the model proposed in this study. We utilize the ordered logit model (Train, 2009) to estimate the preference coefficients based on this survey. Table 1 lists the estimation result. It can be seen that the p-values for the estimates are approximated to zero, which indicates the estimation result is significant with a confidence level of 99%.

The layout of the Munich area used in the following experiments is shown in Fig. 6. This area (about 900 km^2) is divided into 20 zones resulting in $20 \times 19 = 380$ possible OD pairs (internal trips of each zone are ignored). The road traffic demand data partially calibrated with traffic counts collected on May 9th, 2017 (Tuesday) are used. Note, since both public transport and private transport are considered, we need to scale up the road traffic demand (private transport) based on the modal split of the Munich network. In order to mitigate the randomness of the ridesplitting market, we only consider the OD pairs whose travel demand is greater than 100 rather than all OD pairs within the network. This restricts the services to 45 ODs with 7,726 trips in total (we focus on an off-peak period between 5 a.m. to 6 a.m.). The direct trip time and distance of each OD pair are estimated by averaging all trips of the same OD generated by Simulation of Urban MObility (SUMO) (Lopez et al., 2018). To eliminate the stochasticity in simulations, results from 10 replications are averaged. All simulations are implemented at the mesoscopic level through the non-iterative dynamic stochastic user route choice assignment (i.e., automated routing in SUMO).

Fig. 6 Network of the Munich city

We assume the attributes of public transport and private vehicles are given as in Table 2. It is worth noting that the walking time to the station of public transport and the searching time for parking of private car is counted as a part of the waiting time. The determination of attribute values in Table 2 partially refers to Tsiamasiotis et al. (2021), while the attributes of ridesplitting services are inherently decided by the ME model. Moreover, we assume the operating cost per vehicle per hour $\phi = 15$ Euro/h. Other prices imposed externally, such as congestion pricing (de Palma and Lindsey, 2011; Do Chung et al., 2012; Wang et al., 2014; Laval et al., 2015; Cheng et al., 2017) and road pricing (Cramton et al., 2018), are not considered in this study and are left for future work.

Table 2 Attributes of public transport and private vehicles (source: based on Tsiamasiotis et al. (2021)).

Mode	Waiting time (min)	Travel time (min)	Trip fare (Euro)
Public transport	12	$2t_i^d$	$0.3d_i + 1.5$
Private car	5	t_i^d	$0.5d_i + 3$

In order to calculate the market model parameters (A and B), we assume the average detour time and average waiting time of the ridesplitting services in Munich is 30% of the average direct trip time and 4 minutes, respectively, when the vehicle fleet size $\hat{N} = 400$ and the unit price $\hat{p} = 1.00$ Euro/km. The average trip fare then is $\hat{r} = \hat{p}\bar{d}$. Such a market leads to $A = 120.546, B = 0.026$ by using the method present in Appendix A, which will be applied in all hereafter experiments. In practice, one can calibrate the parameters with operation data to characterize the market of interest. Let $\theta = \rho_c = \rho_d = \rho = 1$ in the waiting time model.

When generating the individual trips for calculating the compensations, the standard deviations of waiting time and detour time are set to be one-third of the means. We apply the following compensated utility function to calculate the utility after compensation in this study. For ease of reading, its derivation is omitted here and can be found in Appendix C.

$$V^a = \begin{cases} V & \text{if } V > a \\ -\sqrt{2aV - a^2} & \text{otherwise} \end{cases} \tag{40}$$

As a result, the compensation function is given by

$$c_{i,k} = \begin{cases} 0 & \text{if } V_{i,k} > a \\ \frac{1}{\beta_r}(-\sqrt{2aV_{i,k} - a^2} - V_{i,k}) & \text{otherwise} \end{cases} \tag{41}$$

6.2 Operations under distance-based unified pricing

This section shows the operation performance of ridesplitting services under the distance-based unified pricing method. The operational objectives and endogenous variables are plotted as contour maps in a two-dimensional space formed by the decisions in Fig. 7 and Fig. 8, respectively. Meanwhile, the monopoly optimum (MO) and social optimum (SO) found by the GD algorithm are also marked in the figure.

Fig. 7 shows the iso-profit contours and iso-welfare contours in a two-dimensional space with vehicle fleet size on the x-axis and unit price on the y-axis. As pointed out by Yang and Wong (1998), in particular, if the fleet size is too small, a steady-state equilibrium solution may not exist for a network-based equilibrium model. Accordingly, we can also observe an empty region in the lower left of the figure for the ridesplitting market. Further, it can be seen that the optimal unit price for a monopoly is higher than the optimal unit price at SO, while the MO fleet size is greater than the SO fleet size. Let (N^*_{mo}, p^*_{mo}) and (N^*_{so}, p^*_{so}) denote the coordinates of MO and SO, respectively. Then, $p^*_{mo} > p^*_{so}$ and $N^*_{mo} < N^*_{so}$. This is in accord with our daily understanding. To benefit the public, the services should be operated more widely and cheaply. According to Fig. 7, generally, both profit and welfare first increase with the unit price and fleet size and then decrease. Note that when the unit price is relatively high, the joint influence of decision variables on profit and welfare are similar. When the unit price is relatively small, however, the movements of the two contours become significantly different. It implies that the design of operation strategies should be dedicated specifically to a market with particular consideration of its characteristics and objectives. Moreover, the shape of the contours also verifies the finding that, in the reasonable ranges of decision variables, the local optimum of **P1** or **P2** is also the global optimum. Thus, it is appropriate to apply the GD algorithm to solve them.

Fig. 8a-Fig. 8d depict the contours of ridesplitting demand, seats occupancy rate, network average detour time and network average waiting time, respectively. Let $\lambda, \bar{\tilde{t}}_i$ and $\bar{w}_i$ denote the seats occupancy rate, network average detour time and network average waiting time, then we have

$$\lambda = \frac{\sum_j Q_j t_j}{N n_s}, \bar{\tilde{t}} = \frac{\sum_i Q_i \tilde{t}_i}{\sum_i Q_i}, \bar{w} = \frac{\sum_i Q_i w_i}{\sum_i Q_i} \tag{42}$$

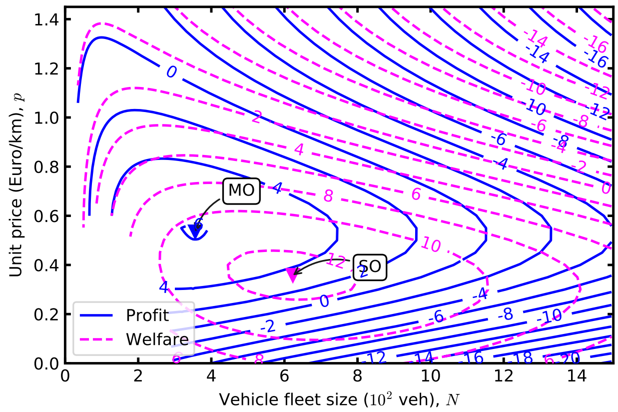
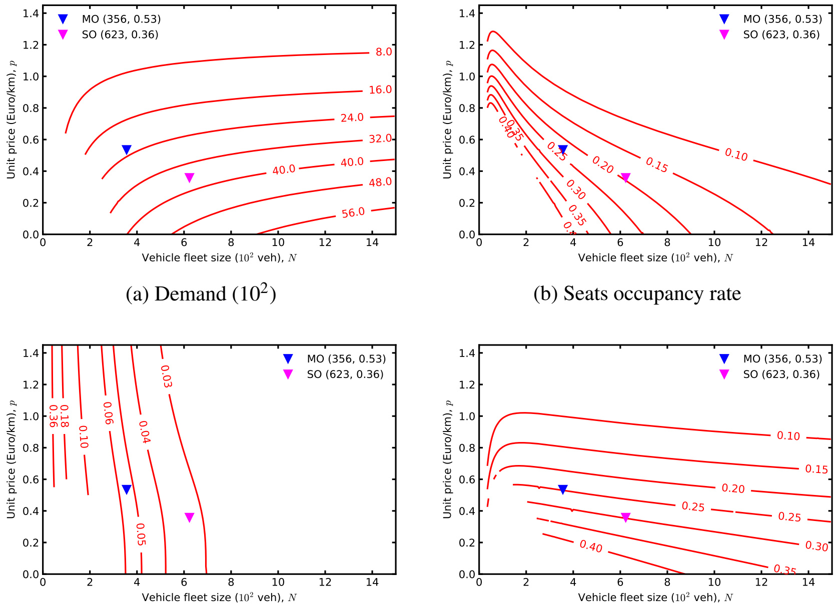


Fig. 7 Profit and welfare in a two-dimensional space of vehicle fleet size and unit price.

Noteworthy, λ reflects the profitability of the ridesplitting service market at a regulated price (Yang and Wong, 1998). Fig. 8b shows that λ decreases with both unit price and vehicle fleet size. Decreasing with the unit price is because a higher price will reduce the passenger demand due to the negative price elasticity. Decreasing with the fleet size is because, despite more vehicles can attract more passengers due to less detour time and waiting time, the induced demand cannot meet the increment of seats supply. Meanwhile, we can see that MO occupancy and SO occupancy are 0.25 and 0.20 (excluding the driver), respectively, resulting in a vehicle occupancy rate of 2.5 persons and 2.2 persons (including the driver), which are more superior compared to the German average of 1.5 persons (van Dender et al., 2013).

Further, it can be seen from Fig. 8c and Fig. 8d that the network average detour time is mainly dependent on the vehicle fleet size. In contrast, the network average waiting time is mainly affected by the unit price. This phenomenon conforms to the assumptions and their formulations. If we assume the direct trip time is the same for all OD pairs (denoted as t^d), then Equation (12) becomes $\tilde{t}_i = At^d/N$. As a result, $\bar{\tilde{t}} = At^d/N$ such that $\bar{\tilde{t}} \propto 1/N$. It indicates that fleet size is the dominant of network average detour time. However, we can also observe that price influence is strengthened when the fleet size becomes larger. The possible reason may be the complicated network structure enhances the heterogeneity of the direct trip time such that ridesplitting demand plays a more important role in determining the detour time. In contrast, as shown in Fig. 8a, ridesplitting demand is primarily dominated by the unit price. In terms of the network average waiting time, we have $w_i \propto Q_i^{\theta}/\sqrt{Nn_s - \sum_j Q_j t_j}$. The influence of N is weakened by the square root operator. Thus it mainly depends on the ridesplitting demand and thus unit price. This relationship would not change after averaging.

Recall that in market assumptions, the unit price is 1.00 Euro/km, the fleet size is 400, and the average waiting time is 4 minutes. However, at MO, for instance, the unit price becomes 0.53 Euro/km, and the waiting time increases to 15 minutes with 356 vehicles. It implies that passengers can bear a longer waiting time to enjoy a cheaper



(a) Demand (10^2)

(b) Seats occupancy rate

(c) Network average detour time (h)

(d) Network average waiting time (h)

Fig. 8 Endogenous variables in a two-dimensional space of vehicle fleet size and unit price.

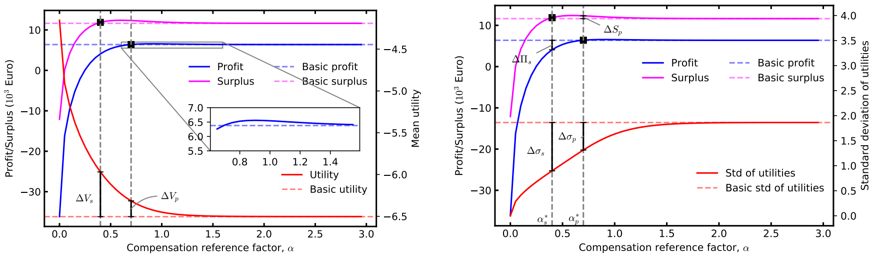
service. We want to mention that the aforementioned system performance measures depend on the market assumptions (A and B), the experiment market/network, and the preference coefficients in the utility function.

6.3 Benefits of the utility-based compensation pricing

To improve the equity and expected LoS, we proposed a utility-based compensation pricing method for ridesplitting services. LoS and equity are represented by the mean and standard deviation of trips utilities, respectively. In this section, the evaluation of market performance under different CRFs (α) is conducted on the basis of the MO (i.e., $N^*_{mo} = 356, p^*_{mo} = 0.53$) operation strategy and the SO operation strategy (i.e., $N^*_{so} = 623, p^*_{so} = 0.36$) separately.

Fig. 9a depicts the profit, social welfare, and mean of utilities under different CRFs on the basis of the MO solution with CRF on the x-axis, profit/welfare on the left y-axis, and mean utility on the right y-axis. Clearly, profit and welfare increase with CRF and end up with the respective basic values, where basic values are the values at the equilibrium of the MO solution for the unified pricing scenario. It can be seen that the peaks of the surplus and profit curves are higher than the basic ones. It implies that the proposed compensation pricing approach can benefit both profit and welfare if disregarding the seek of improving LoS and equity. The maximum profit and maximum welfare increase by 2.9% (from 6,378 Euro to 6,560 Euro) and 6.5%

(from 11,630 Euro to 12,388 Euro), respectively. Denote the x coordinate of the first meeting point between the basic profit (dashed blue) and the profit curve (solid blue) as α_p^*. Similarly, denote the x coordinate of the first meeting point between the basic surplus and the surplus curve as α_s^*. Then, we have $\alpha_s^* < \alpha_p^*$. Since the curve of mean utility is monotonically decreasing, so $\Delta V_s > \Delta V_p$, where ΔV_s and ΔV_p denote the improvement of LoS under α_s^* and α_p^*, respectively. Likewise, compensation under α_s^* can also improve service equity (utility variance) more than under α_p^*, i.e., $\Delta\sigma_s > \Delta\sigma_p$, as shown in Fig. 9b.



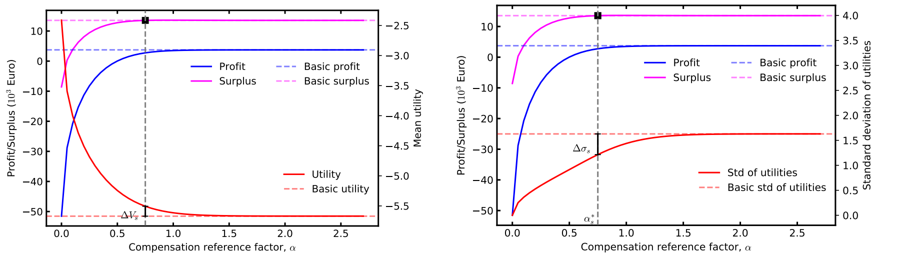
(a) LoS (mean of utilities) (b) Equity (standard deviation of utilities)

Fig. 9 Performance of the utility-based compensation pricing method under different CRFs based on the MO operation strategy.

Further, as indicated in Fig. 9b, implementing compensation under α_s^* will lead to a reduction of profit by $\Delta\Pi_s$. This provides a reference to the relevant department regarding the development of the subsidy policy. To maximize the LoS and equity of ridesplitting services without sacrificing any profit and social welfare, the service operator implementing the compensation pricing approach should be subsidized with an amount of at least $\Delta\Pi_s$ (the analysis regarding subsidy is detailed in Section 7.2.1). Suppose no subsidies are possible under α_p^*. In that case, one can not only improve the LoS and equity (though the improvement is shrunken compared to the case under α_s) but also contribute to additional welfare of ΔS_p (5.9%, from 11,630 Euro to 12,320 Euro). One can even see an increase in both profit and welfare in the range between α_p and the second meeting point between the basic profit and the profit curve. It is worth noting that the benefit to LoS and equity would be impaired with the increase of α. Table 3 provides the influence on the system endogenous variables when applying compensation pricing under the two mentioned meeting CRF points.

Table 3 The performance of compensation pricing on the system endogenous under MO.

CRF	Ridesplitting demand	Seats occupancy rate	Waiting time	Detour time
α_p^*	10.3%	14.5%	15.0%	6.1%
α_s^*	33.6%	41.0%	48.5%	9.0%



(a) LoS (mean of utilities) (b) Equity (standard deviation of utilities)

Fig. 10 Performance of the utility-based compensation pricing method under different CRFs based on the SO operation strategy.

Fig. 10 illustrates the performance of the proposed compensation approach in the SO operation scenario. It is worth noting that the first meeting point between the basic profit and the profit curve does not exist. That is, a subsidy is necessary for the operator to implement the compensation method in this case. Otherwise, it will produce a profit loss compared to the unified pricing case. Likewise, there is also nearly no increase in the maximum welfare (only increase about 0.6%). Moreover, the improvement of LoS and equity under α_s^* is diminished compared to that in the case of MO. Therefore, we state that the proposed compensation pricing method is more beneficial for a market aiming at maximizing profit. However, to a certain extent, this also implies the inefficiency of a monopoly market.

7 Discussion

7.1 Market equilibrium

7.1.1 Model parameters

The model parameters A and B are critical in establishing a reliable ME model for ridesplitting market analysis. Note that A and B are the exogenous parameters of the expected detour time and waiting time estimation models. We apply the method described in Appendix A with some approximations to calculate them for a synthesized market. It is worth mentioning that an appropriate calibration procedure for A and B using the real data of experienced waiting and detour times is required (part of ongoing research). Besides, other possibilities of calibrating these parameters with a traffic simulation model are also presented in Fig. 11 (for cases of absence of operational data).

7.1.2 Fleet deployment

In the ME model, we assume the expected detour time among different OD pairs to be proportional to the direct trip time of the corresponding OD. However, the situation is more complicated in practice (for example, it should also relate to how the passenger demand is distributed over the network) and requires further investigation through either empirical experiment or analytical derivation. Also, the expected waiting time model only accounts for the spatial difference of vehicle allocations and the potential effect of demand on supply attraction. A more well-designed model is required to fit the spatial difference and the temporal elasticity of the ridesplitting supply and demand. Further, as discussed in Fagnant and Kockelman (2018), fleet operations are also a vital factor to the system profitability and with significant consequences on customer experience. The ME model demonstrates that the fleet deployment strategy should also somehow affect the distribution of ridesplitting demand in the long-term operation by influencing the detour and waiting times. Thus, this is rather a complex topic that requires more effort to parse.

Apart from the further analytical modeling improvements in the ME model, another reliable solution to better explore an optimal deployment strategy is integrating the proposed modeling framework into a traffic simulation platform for experimental implementation. Modeling the ridesplitting market with a dynamic traffic simulator provides the opportunity to model individual vehicles with opportunities to implement fleet deployment strategies and have realistic service attributes of waiting and detour times at the expense of higher computational effort.

7.1.3 Simulation integration

The ME model approximates the service attributes or more indirectly performs the mode choice of ridesplitting services analytically. However, the mode choice step in a ridesplitting simulation platform is somewhat computationally expensive, requiring equilibrium/convergence to get a specific set of service attributes for any change in exogenous variables (Liu et al., 2019). Integrating a ME model in a ridesplitting simulation platform can help remove the required computational effort for mode choice. In return, the ME model approximation can also be further improved by a feedback calibration loop of its model parameters, based on the experienced service attributes from the simulation (Fig. 11), as suggested in section Section 7.1.1.

7.2 Subsidy schemes based on utility-based compensation pricing

7.2.1 Subsidy schemes

Agatz et al. (2012) suggested that, in order to improve the density of ridesplitting riders, local governments need to subsidize ridesplitting initiatives. Utility-based compensation pricing methods provide opportunities to improve the effectiveness of such subsidy schemes. Ridesplitting services, by their nature, cannot promise equity in perceived trip LoS among different passengers and may have negative impacts due to uncertainty (Guan et al., 2019a). Subsidies provided through utility-based compensation pricing can help improve equity and average utilities significantly (as shown in Fig. 9 & Fig. 10). In addition, the improvement in average utilities also attracts more passenger demand contributing to a higher mode share of the service. Fig. 12 depicts

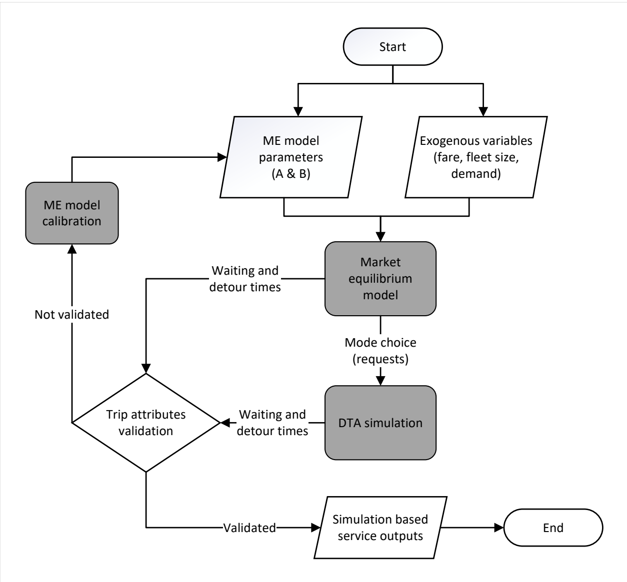
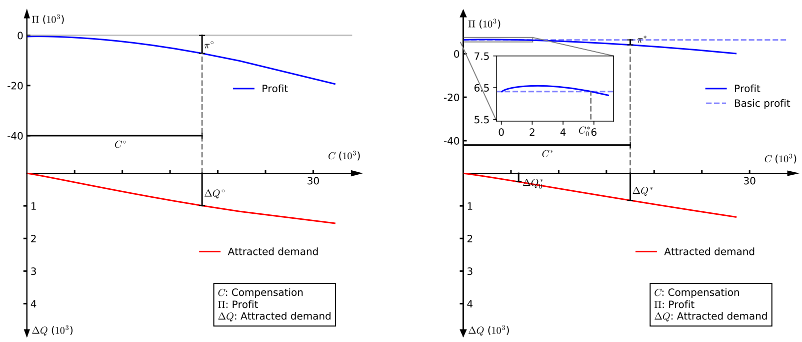


Fig. 11 Flow chart for integrating market equilibrium with DTA simulation

the relation between profit, compensation, and attracted demand for two different situations: (i) the start-up phase of operation, when both operator and government aim to promote the ridesplitting services (assuming 40% people are exposed to the service); (ii) the mature phase of operation when the operator wants to maintain operation afloat and the government intends to promote the service.



(a) Start-up operation phase (b) Stable operation phase

Fig. 12 Comparing compensation, profit and attracted demand in MO operation strategy

7.2.2 Demand management

Ridesplitting services require critical mass/density of passengers for sustainable operations (Furuhata et al., 2013). From Fig. 12, we can see that, at both the start-up stage and the mature stage, the operator can perform demand management efficiently under the proposed pricing method by implementing different compensation strategies. On the other hand, the government can also manage both the ridesplitting demand and road travel demand by adjusting the subsidy scheme. For example, if the government subsidizes an amount of π to the service operator (assuming the subsidies are invested into compensations), the ridesplitting demand will increase by ΔQ. On the contrary, private vehicles on the road and the demand for public transport will decrease correspondingly. As such, apart from capitally investing in the construction and expansion of the road network or public transport, subsidizing ridesplitting services could provide a relatively inexpensive way to enhance the efficiency of urban transportation systems (Agatz et al., 2012).

7.2.3 Dynamic pricing

Obviously, both the distance-based unified pricing and the utility-based compensation pricing introduced in this paper are not dynamic (i.e., the compensations are estimated periodically for the trips finished within the interval) due to ME-based modeling. A realistic implementation of such pricing (e.g., in a DTA simulation) would require dynamic compensation (i.e., a passenger fare is compensated upon trip termination). Dynamic compensation pricing can be done, setting α as a dynamic compensation control variable, and profit, attracted demand and average LoS as control updating variables. Recall that the prices imposed externally (such as congestion pricing and road pricing) are not considered in this paper and can have impacts when the trip fare is determined dynamically because these external prices are always only valid for a specific part of the network (e.g., city center) within a specific time period in a day. Consequently, it is appropriate to consider them when extending the pricing method to be dynamic.

8 Conclusions

This paper focuses on the inequity problem existing among individual trips in the ridesplitting markets. We construct a network-based market equilibrium model to estimate the market responses to the operator's decision on trip fare and vehicle fleet size. In this model, the complicated relationships between the decisions and system endogenous variables (e.g., ridesplitting demand, waiting time, detour time) in ridesplitting markets are synthesized into a simultaneous equations system. A Gradient Descent algorithm is applied to find the monopoly optimum and social optimum under the distance-based unified pricing method. The result shows that the MO unit price is higher than the SO unit price, while the MO fleet size is smaller than the SO fleet size. We also show that network average detour time and network average waiting time are mainly influenced by the vehicle fleet size and the unit price, respectively, while the seats occupancy rate is dependent on both of them. In addition, limitations, possible extensions, and applications of market equilibrium, i.e., its calibration, usage for service mode choice, and integration in the DTA simulation platform, are also explored.

A utility-based compensation pricing method is developed to improve equity and average LoS on the basis of the unified pricing method. With this method, trips with a utility below a threshold (i.e., CRP) are compensated based on a predefined compensated utility function (describing the relationship between the utility before and after compensation). Note that, LoS and equity of services are represented by the mean of and variance of trips' utilities, respectively. The result shows that the improvement of LoS and equity is more pronounced when the compensation approach is utilized after the MO solution compared with when it is after the SO solution. In the former case, we can even see an increase in the maximum profit and welfare by applying this compensation method with a specific range of CRFs. Further, its implementation in different domains, i.e., for more effective subsidy schemes and dynamic pricing, is also discussed.

Author contributions

Conceptualization: Qing-Long Lu, Moeid Qurashi, Constantinos Antoniou; Methodology: Qing-Long Lu, Moeid Qurashi; Software: Qing-Long Lu; Formal analysis: Qing-Long Lu, Moeid Qurashi, Constantinos Antoniou; Investigation: Qing-Long Lu, Moeid Qurashi; Visualization: Qing-Long Lu; Project administration: Moeid Qurashi; Writing - original draft: Qing-Long Lu; Writing - review and editing: Moeid Qurashi, Constantinos Antoniou; Funding acquisition: Constantinos Antoniou; Supervision: Constantinos Antoniou.

Conflict of interest

The authors declare that they have no conflict of interest.

A Calibration of model parameters

The estimation accuracy of expected detour time and expected waiting time can significantly affect the effectiveness of the proposed ridesplitting equilibrium model. Therefore, it is critical to provide plausible A and B in the detour and waiting time models. Remarkably, A and B would be different for different markets. Concisely, A and B can capture the particular traits of the market and thus lead to a reliable market model for relevant analyses.

In this section, we introduce an approximating method to calculate A and B for a market given the operation data. Suppose that we know the vehicle fleet size $\hat{N}$, average trip fare $\hat{r}$, average detour time $\hat{\bar{t}}$, and average waiting time $\hat{w}$ from the operation of the market of interest. Assume the probabilities of choosing ridesplitting are the same for all OD pairs in the network, according to the MNL model, we can simply get

$$\hat{P} = \frac{e^{\hat{V}_{rs}}}{\hat{\mu} + e^{\hat{V}_{rs}}} \tag{43}$$

where $\hat{V}_{rs} = \beta_t(\bar{t}^d + \hat{\tilde{t}}) + \beta_w \hat{w} + \beta_r \hat{r}$. And $\hat{\mu} = \sum_{j\in\{\mathbb{M}-rs\}} e^{\hat{V}_j}$ is the sum of the exponential of utilities of other transportation options.

Recall that the expected waiting time of OD i is estimated by

$$w_i = \frac{B_i Q_i^\theta}{\Omega_i \sqrt{Nn_s - \sum_j Q_j t_j}} \tag{44}$$

Assume the network is homogeneous: (i) the expected waiting time is the same across the network ($w_i = \hat{w}$), (ii) the ridesplitting demand is the same for all OD pairs ($Q_i = \hat{Q}_i, \Omega_i = 1$), and (iii) the expected detour time is the same across the network ($\tilde{t}_i = \hat{\tilde{t}}$). Then, $Q_i = \hat{P}\sum_i D_i/n_z$ and $\sum_j Q_j t_j = \hat{P}\sum_j D_j(\hat{\tilde{t}} + t_j^d)$, resulting in

$$B = \frac{\hat{w}\sqrt{\hat{N}n_s - \hat{P}\sum_j D_j(\hat{\tilde{t}} + t_j^d)}}{(\hat{P}\sum_i D_i/n_z)^\theta} \tag{45}$$

On the other hand, recall that the detour time model is given by

$$\tilde{t}_i = \frac{A t_j^d \sum_j Q_j t_j^d}{\bar{t}^d N \sum_j Q_j} \tag{46}$$

Similarly, we approximate $t_j^d \approx \bar{t}^d, \sum_j Q_j t_j^d \approx \hat{P}\sum_j D_j t_j^d, \sum_j Q_j \approx \hat{P}\sum_j D_j$, then we can get,

$$A = \frac{\hat{\tilde{t}}\hat{N}\sum_j D_j}{\sum_j D_j t_j^d} \tag{47}$$

Equation (45) and Equation (47) provide a reliable approximated value of B and A, respectively. Note that sometimes we may still need to tune the result from this calculation procedure to make the model with the same input conditions (i.e., vehicle fleet size and price) result in a similar operation result.

B Partial derivatives of detour time and waiting time

As shown in Section 4.1 and Section 4.2, the derivatives of the objectives are determined by the derivatives of detour time and waiting time.

Let $t'_{i|p}$ and $t'_{i|N}$ denote the derivatives of t_i with respect to p and N, respectively. And let $w'_{i|p}$ and $w'_{i|N}$ denote the derivatives of w_i with respect to p and N, respectively. Recall that $\tilde{t}_i = A_i \sum_j Q_j t_j^d/(N\sum_j Q_j)$, and since $t_i = t_i^d + \tilde{t}_i$, then $t'_{i|p} = \tilde{t}'_{i|p}$, thus the derivative of t_i with respect to p can be calculated as

$$t'_{i|p} = \frac{A_i}{N}\left[\frac{1}{\sum_j Q_j}\sum_j \frac{\partial Q_j}{\partial p} t_j^d - \frac{\sum_j Q_j t_j^d}{(\sum_j Q_j)^2}\sum_j \frac{\partial Q_j}{\partial p}\right] \tag{48}$$

where

$$\frac{\partial Q_j}{\partial p} = D_j P_j (1 - P_j)(\beta_t t'_{j|p} + \beta_w w'_{j|p} + \beta_r d_j) \tag{49}$$

It means Equation (48) is a linear combination of $t'_{i|p}$ and $w'_{i|p}$.

On the other hand, since $w_i = B_i Q_i^\theta/(\Omega_i\sqrt{Nn_s - \sum_j Q_j t_j})$, so

$$w'_{i|N} = \frac{B_i \theta Q_i^{\theta-1}}{\Omega_i\sqrt{Nn_s - \sum_j Q_j t_j}}\frac{\partial Q_i}{\partial p} - \frac{B_i Q_i^\theta}{\Omega_i^2\sqrt{Nn_s - \sum_j Q_j t_j}}\frac{\partial \Omega_i}{\partial p} - \frac{B_i Q_i^\theta}{2\Omega_i\sqrt{(Nn_s - \sum_j Q_j t_j)^3}}\frac{\partial \sum_j Q_j t_j}{\partial p} \tag{50}$$

where $\Omega_i = n_z \sum_{j\in\mathbb{Z}_i} Q_j/(\sum_{k\in\mathbb{Z}}\sum_{j\in\mathbb{Z}_k} Q_j)$, and

$$\frac{\partial \Omega_i}{\partial p} = \frac{n_z}{\sum_{k\in\mathbb{Z}}\sum_{j\in\mathbb{Z}_k} Q_j}\sum_{j\in\mathbb{Z}_i}\frac{\partial Q_j}{\partial p} - \frac{n_z\sum_{j\in\mathbb{Z}_i} Q_j}{(\sum_{k\in\mathbb{Z}}\sum_{j\in\mathbb{Z}_k} Q_j)^2}\sum_{k\in\mathbb{Z}}\sum_{j\in\mathbb{Z}_k}\frac{\partial Q_j}{\partial p} \tag{51}$$

Thus, Equation (50) is also a linear combination of $t'_{i|p}, w'_{i|p}, \forall i$.

Combining Equation (48) and Equation (50) in terms of different i, we can get a linear equations system with $2n_z$ unknowns ($t'_{i|p}, w'_{i|p}, \forall i \in \mathbb{Z}$) and $2n_z$ equations (linear combinations). For a given p and N (at a specific iteration of the algorithm), this linear equations system is tractable via linear algebra.

Similarly, we can obtain the derivatives with respect to N as below.

$$t'_{i|N} = \frac{A_i}{N\sum_j Q_j}\sum_j \frac{\partial Q_j}{\partial N} t_j^d - \frac{A_i \sum_j Q_j t_j^d}{(N\sum_j Q_j)^2}(\sum_j Q_j + N\sum_j \frac{\partial Q_j}{\partial N}) \tag{52}$$

$$w'_{i|N} = \frac{B_i \theta Q_i^{\theta-1}}{\Omega_i \sqrt{Nn_s - \sum_j Q_j t_j}} \frac{\partial Q_i}{\partial N} - \frac{B_i Q_i^{\theta}}{\Omega_i^2 \sqrt{Nn_s - \sum_j Q_j t_j}} \frac{\partial \Omega_i}{\partial N} - \frac{B_i Q_i^{\theta}}{2\Omega_i \sqrt{(Nn_s - \sum_j Q_j t_j)^3}} \left(n_s - \frac{\partial \sum_j Q_j t_j}{\partial N}\right) \tag{53}$$

where

$$\frac{\partial \Omega_i}{\partial N} = \frac{n_z}{\sum_{k\in\mathbb{Z}}\sum_{j\in\mathbb{Z}_k} Q_j}\sum_{j\in\mathbb{Z}_i}\frac{\partial Q_j}{\partial N} - \frac{n_z \sum_{j\in\mathbb{Z}_i} Q_j}{(\sum_{k\in\mathbb{Z}}\sum_{j\in\mathbb{Z}_k} Q_j)^2}\sum_{k\in\mathbb{Z}}\sum_{j\in\mathbb{Z}_k}\frac{\partial Q_j}{\partial N} \tag{54}$$

$$\frac{\partial Q_j}{\partial N} = D_j P_j (1-P_j)(\beta_t t'_{j|N} + \beta_w w'_{j|N}) \tag{55}$$

Analogously, the linear equations system consists of Equation (52) and Equation (53) ($\forall i \in \mathbb{Z}$) can be solved via linear algebra.

C Derivation for the compensated utility function

By applying the utility-based compensation method, we do compensations for the trips whose utility is less than a predefined threshold a ($a < 0$). In this study, we assume the form of the compensated utility function as

$$f(x) = l\sqrt{-x+b} \tag{56}$$

with following mild assumptions.

Assumption 5 *$f(x)$ is continuous and smooth on (a, a), such that: 1) $f(a) = a$; 2) $f'(a) = 1$.*

Obviously, the function in Equation (56) satisfies the properties described in Section 5. Based on Assumption 5, we have

$$\begin{cases} f(a) = l\sqrt{-a+b} = a & (57) \\ f'(a) = -\frac{1}{2}l(-a+b)^{-\frac{1}{2}} = 1 & (58) \end{cases}$$

As utilities are negative, i.e., $a < 0, f(a) < 0$, we then know $l < 0$ from Equation (57). After some straightforward work, we arrive at

$$f(x) = -\sqrt{2ax - a^2} \tag{59}$$

Consequently, let V^a denote the utility after compensation, then the full formulation of the compensated utility function is given by

$$V^a = \begin{cases} V & \text{if } V > a \\ -\sqrt{2aV - a^2} & \text{otherwise} \end{cases} \tag{60}$$

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