

1 **MODELING CROWDEDNESS AT PUBLIC TRANSPORT STATIONS DURING**  
2 **SPECIAL EVENTS: A COMPARATIVE STUDY OF ELEVEN CITIES**

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6 **Wanrong Hu**

7 Chair of Transportation Systems Engineering, Technical University of Munich, Germany

8 Email: diana.hu@tum.de

9

10 **Qing-Long Lu, Corresponding Author**

11 Chair of Transportation Systems Engineering, Technical University of Munich, Germany

12 Email: qinglong.lu@tum.de

13

14 **Ningkang Yang**

15 Chair of Transportation Systems Engineering, Technical University of Munich, Germany

16 Email: ningkang.yang@tum.de

17

18 **Constantinos Antoniou, Ph.D.**

19 Chair of Transportation Systems Engineering, Technical University of Munich, Germany

20 Email: c.antoniou@tum.de

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**1 ABSTRACT**

2 Special events are the primary trigger for the abnormal crowdedness patterns present in public  
3 transport (PT) systems. Accurately predicting these changes in crowdedness can enhance PT  
4 scheduling and crowd management, making it a crucial aspect of public transport demand manage-  
5 ment (PTDM). This study aims to forecast short-term crowdedness at PT stations during planned  
6 special events using publicly available opportunistic data. More specifically, we take football  
7 matches as an example of special events, and use Google Popular Times (GPT) as opportunist-  
8 tic data given its extensive spatiotemporal coverage in urban areas. We conduct a comparative  
9 analysis involving 15 football clubs from 11 different cities. We develop a graph-based neural  
10 network augmented with an attention mechanism and a positional embedding-enhanced temporal  
11 convolutional network to predict crowdedness at metro stations. Additionally, we design a spe-  
12 cific event indicator to capture the unique temporal characteristics associated with special events.  
13 The experimental results demonstrate that our proposed model effectively forecasts crowdedness at  
14 metro stations, with the inclusion of the event indicator significantly enhancing prediction accuracy  
15 during special events.

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17 *Keywords:* Crowdedness patterns, special events, opportunistic data, comparative analysis, graph  
18 neural network

## 1 INTRODUCTION

2 Special events, whether planned or unplanned, such as the Olympics, football games, concerts, and  
3 other large gatherings, play a significant role in our lives and culture. However, these events also  
4 cause an abnormal increase in traffic demand, potentially decreasing service levels or even leading  
5 to network failures(1). Urban rail transport systems are pivotal in addressing the challenges posed  
6 by urban sprawl and suburbanization, offering substantial social, economic, and environmental  
7 benefits globally.

8 Effective planning and management are crucial to handle the demand changes triggered  
9 by special events. Short-term forecasting of passenger flow, typically predicting trends within  
10 a 5-minute to 1-hour window, is integral to urban rail transit management and crowd regula-  
11 tion. Accurate short-term forecasts enable operators to optimize service schedules, plan station  
12 crowd regulation measures, and enhance passenger information systems. These forecasts ensure  
13 the transportation supply aligns precisely with passenger demand, facilitating emergency response  
14 planning. Therefore, evaluating the evolution of crowd patterns and forecasting passenger flow  
15 during special events is essential for effectively developing strategies to address these challenges.

16 Rail passenger forecasting is a vital topic in transportation engineering with wide-ranging  
17 applications. It can be broadly divided into long-term forecasting, which usually spans years or  
18 months and guides the planning and construction of rail systems, and short-term forecasting, which  
19 is the focus of our research. Depending on the data format used, these forecasting problems can be  
20 classified into various types, including time series, grid, graph, and hybrids like temporal-spatial  
21 data. Traditional statistical models such as autoregressive integrated moving average (ARIMA) (2)  
22 and its variants, including seasonal autoregressive integrated moving average (SARIMA) (3) and  
23 generalized autoregressive conditional heteroscedasticity (GARCH) (4), have been widely used in  
24 traffic demand prediction. More recently, robust principal component analysis (RPCA) (5) has also  
25 been employed to address some limitations of ARIMA models.

26 With the advent of AI technology, deep learning models have gained popularity in trans-  
27 portation system research due to their efficiency and accuracy in handling large-scale data and  
28 capturing nonlinear relationships. Despite the large number of models developed for traffic flow  
29 forecasting in recent years, they can be categorized into convolutional neural network (CNN)-based  
30 models, recurrent neural network (RNN)-based models, Graph neural network (GNN)-based mod-  
31 els, and various hybrids or variants. Most recent studies in short-term traffic state forecasting show  
32 that while statistical models primarily focus on temporal dependencies, deep learning models excel  
33 by incorporating both temporal and spatial features (6).

34 Despite the growing interest in deep learning models for short-term rail transit system pas-  
35 senger forecasting, particularly during special events, most studies rely on data from automated  
36 fare collection systems, which are not always available. For example, there are no such systems  
37 for public transport in most cities in Germany. Our study addresses these gaps using Google Popu-  
38 lar Times (GPT) data. Specifically, we collected popularity data on public transport stations across  
39 ten cities and 17 metro lines to capture the temporal-spatial characteristics of passenger crowding.  
40 We employ a graph-based neural network, the attention with a positional embedding enhanced  
41 temporal convolutional network (APT-GCN) to predict crowdedness at metro stations, specifically  
42 focusing on the patterns under special events. Our model is trained and tested on this dataset to  
43 analyze crowdedness patterns across various public transport networks.

44 The rest of the paper is structured as follows: following this introduction, we detail the  
45 process of data collection and present our model in the Methodology section. Next, we train the

1 proposed model and discuss testing results. Finally, the paper concludes with a summary of our  
2 findings.

### 3 **DATA COLLECTION AND PROCESSING**

4 In this section, we present a comprehensive overview of the data collection and processing. The  
5 dataset is bifurcated into three components: special events data, public transport network data and  
6 Google Popular Times (GPT) data.

7       The special events dataset is first elaborated upon, where we define and describe the spe-  
8 cific events considered in this study. These events are contextualized within the public transport  
9 network and correlated with transit patterns. The second component pertains to the spatial domain,  
10 where the topological relationships of stations along transit lines are collected. Lastly, the tem-  
11 poral domain is addressed. The raw GPT data were collected from 428 public transport stations  
12 across 17 transit lines in ten cities, spanning nearly four months. Following the description of the  
13 GPT data, we performed data analysis and imputation. We also construct a new dummy feature to  
14 encapsulate the special event information.

#### 15 **Special Events Dataset**

16 Special events, such as concerts, sports events, and festivals, significantly impact public transporta-  
17 tion usage and passenger flow. We selected football matches as a representative type of special  
18 event for our study. Football, being one of the most popular sports globally, often attracts large  
19 audiences, enriching people’s lives but also exerting significant pressure on public transport sys-  
20 tems. Our study focuses on 15 clubs from top European football leagues and the public transport  
21 routes passing through their home stadiums. Table 1 provides an overview of the basic information  
22 about these clubs and their associated transit lines. Stops represent the stations associated with  
23 the stadiums and are typically identified using transportation information provided on the clubs’  
24 official websites, where line ID is a unique identifier for each public transport line.

25       Naturally, the various leagues and cup matches hosted at these stadiums are considered our  
26 special events. Within the time frame of our data collection, 122 such events occurred. We use the  
27 kickoff time of each match as the event start time and consider two hours post-kickoff as the event  
28 end time, accounting for the typical duration of a football match. Table 2 illustrate an example of  
29 these events.

#### 30 **Public Transport Network Topology**

31 For the urban rail transit lines involved in the aforementioned special events, we collected their  
32 network maps to establish the spatial topological structures. These structures can be categorized  
33 into three types: linear, circular, and branching. Stations that are spatially adjacent are defined as  
34 those that are connected directly by a single segment of the rail line without any intermediate stops.

#### 35 **Google Popular Time**

##### 36 *GPT data properties*

37 GPT data is provided for each Point of Interest (POI). The raw data from GPT includes historical  
38 popularity, current popularity, visit duration, and wait time estimates.

- 39       • Current Popularity: This indicates the crowd level at a given POI at the current time.
- 40       • Historical Popularity: This is based on average popularity over the past several weeks,  
41       providing a popularity value for each hour of each day of the week, resulting in 7 *times*

**TABLE 1:** Overview of Cities, Public Transport Lines, and Stadiums

City	Line ID	Stops	Football Club
Munich	U6	Fröttmanning	Bayern Munich
Berlin	S3	Berlin-Köpenick	1.FC Union Berlin
Dortmund	U45, U46	Westfalenhallen	Borussia Dortmund
Köln	Tram 1	RheinEnergieStadion	FC Köln
Augsburg	Tram 3, Tram 8	Augsburg WWK ARENA	FC Augsburg
Madrid	Line 10	Santiago Bernabéu	Real Madrid
Madrid	Line 7	Estadio Metropolitano	Atletico Madrid
Madrid	Line 12	Los Espartales	Getafe
Madrid	Line 1	Portazgo	Rayo Vallecano
London	Piccadilly	Arsenal	Arsenal
London	District	Fulham Broadway	Chelsea
Newcastle	Yellow	St James	Newcastle United
Marseille	M2	Sainte-Marguerite Dromel	Olympique de Marseille
Copenhagen	M3	Trianglen St.	F.C. Copenhagen
Lisbon	Blue	Colégio Militar/Luz	Benfica

**TABLE 2:** An Example of Special Events Data

City	Line	Event	Start Time	End Time
Copenhagen	M3	FC Copenhagen-Manchester City	Feb 13, 21:00	Feb 13, 22:55
Lisbon	Blue	Benfica-Toulouse	Feb 15, 21:00	Feb 15, 22:58
Köln	Tram 1	Köln-Bremen	Feb 16, 20:30	Feb 16, 22:27
Madrid	Line 7	Atletico Madrid-Las Palmas	Feb 17, 14:00	Feb 17, 15:51
London	Yellow	Newcastle-Bournemouth	Feb 17, 16:00	Feb 17, 17:58
Lisbon	Blue	Benfica-Vizela	Feb 18, 19:00	Feb 18, 20:58
Marseille	M2	Marseille-Shakhtar	Feb 22, 21:00	Feb 22, 22:59

- 1           24 data points. It shows how crowded the location typically is during different times of  
2           the day.
- 3           • Visit Duration: This data shows how much time customers typically spend at the specific
  - 4           POI, estimated based on patterns of customer visits over the last several weeks.
  - 5           • Wait Time Estimates: This shows how long a customer would have to wait before receiv-
  - 6           ing service during different times of the day.
- 7           We extract current popularity as the primary indicator of station crowding. Additionally, we or-
- 8           ganize other data points as potential features for future use, providing varies models with more
- 9           comprehensive information for analysis.

1 *Data quality analysis*

2 We collected GPT data for the aforementioned metro stations from February 13, 2024, to June 4,  
 3 2024, resulting in 4617 timestamps. Despite our best efforts, GPT data is not always available at  
 4 all stations and times. Data collection failures may stem from the inherent unavailability of data or  
 5 from unavoidable issues like network instability during the collection period. Therefore, we first  
 6 analyze the collected data to provide a basis for subsequent data processing.

7 Our analysis is conducted at the level of each subway line and station. Taking Madrid Metro  
 8 Line 1 as an example, Figure 1 shows the statistics of popularity values at various stations during  
 9 the data collection period, and Figure 2 illustrates the distribution of data points collected over a  
 10 week. Given our setting of collecting data from all stations every 30 minutes, there should ideally  
 11 be 336 data points per week. Stations with abnormal data, poor quality, or significant missing  
 12 values, such as Atocha station in this instance, are removed from the dataset. Subsequently, we  
 13 proceed with data imputation to address the missing data issue.

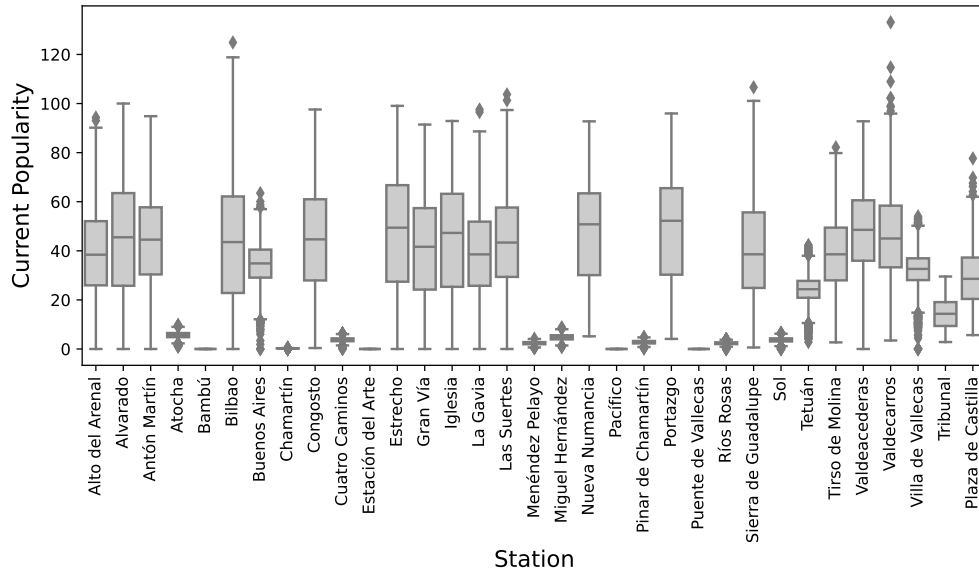


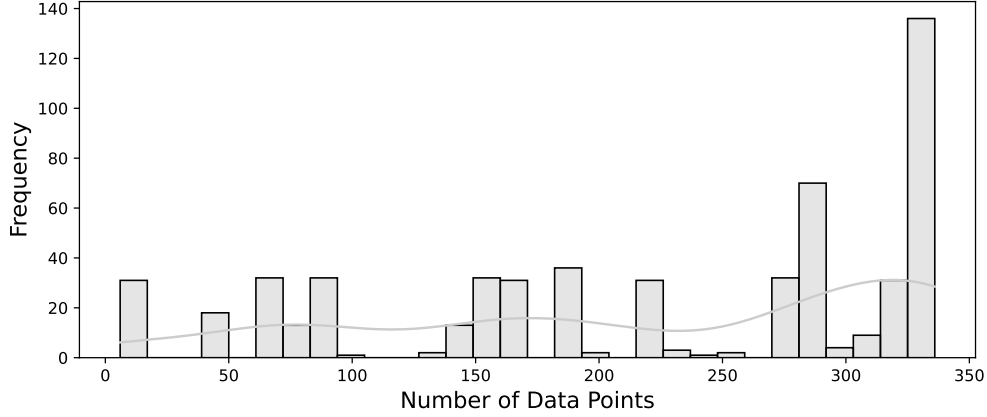
FIGURE 1: GPT popularity value distribution of stations in Madrid Line 1

14 *Data imputation with Temporal Regularized Matrix Factorization (TRMF)*

15 In this section, we introduce the data imputation algorithm that is applied to complete the GPT  
 16 dataset. Our problem can be defined as a random missing spatial-temporal data problem, where  
 17 each sensor loses observations completely at random. To address this issue, TRMF is employed.  
 18 This model incorporates temporal dependencies into the matrix factorization process(7). Temporal  
 19 dependencies are represented by  $\mathbf{x}_i$ ,

$$20 \quad x_t \approx \sum_{l \in \mathcal{L}} \theta_l \odot x_{t-l} \quad (1)$$

21 Then,



**FIGURE 2:** Number of collected data points of stations in Madrid Line 1

$$1 \quad \mathcal{R}_{\text{AR}}(X | \mathcal{L}, \Theta, \eta) = \frac{1}{2} \sum_{t=l_d+1}^f \left( x_t - \sum_{l \in \mathcal{L}} \theta_l \odot x_{t-l} \right)^\top \left( x_t - \sum_{l \in \mathcal{L}} \theta_l \odot x_{t-l} \right) + \frac{\eta}{2} \sum_{t=1}^f x_t^\top x_t \quad (2)$$

2 where  $\mathcal{L} = \{l_1, l_2, \dots, l_d\}$  is a lag set, in our task is setting to  $\mathcal{L} = \{1, 2, 48\}$ , and  $\theta_l \in \mathbb{R}^r, \forall l$  is  
 3 weight to decide autoregressive (AR). Thu the TRMF is to solve,

$$4 \quad \min_{W, X, \Theta} \frac{1}{2} \sum_{(i,t) \in \Omega} \left( y_{it} - w_i^\top x_t \right)^2 + \lambda_w \mathcal{R}_w(W) + \lambda_x \mathcal{R}_{\text{AR}}(X | \mathcal{L}, \Theta, \eta) + \lambda_\theta \mathcal{R}_\theta(\Theta) \quad (3)$$

5 where  $\mathcal{R}_w(W) = \frac{1}{2} \sum_{i=1}^m w_i^\top w_i$  and  $\mathcal{R}_\theta(\Theta) = \frac{1}{2} \sum_{l \in \mathcal{L}} \theta_l^\top \theta_l$  are regularization terms.

6 Consequently, a spatial-temporal popularity data with a 30-minute interval based on GPT  
 7 was obtained by applying data imputation.

8 In summary, this section sets the groundwork for understanding the data-related approaches  
 9 employed in our study, providing a detailed account of the dataset's structure, collection, and  
 10 enhancements, ensuring the reliability of our dataset for subsequent analysis.

## 11 METHODOLOGY

### 12 Problem Definition

13 The primary objective of this study is to analyze the crowding patterns at public transport stations  
 14 during special events and to forecast future crowdedness based on historical data. Crowdedness is  
 15 quantified using the popularity data derive from Google Popular Times, as previously mentioned.  
 16 The study is divided into two main tasks. The first task involves a cross-analysis of the crowding  
 17 patterns along public transport lines in different cities to identify similarities and differences. The  
 18 second task is treated as a short-term traffic forecasting problem. This involves proposing an  
 19 attention with positional embedding enhanced temporal convolutional network (APT-GCN) model  
 20 to predict the future popularity at each station based on a 4-hour historical popularity data.

21 **Definition 1** Public Transport Network  $G$ . A public transport network is described as an  
 22 unweighted, undirected graph  $G = (V, E)$ , representing the inherent typologies of public transport  
 23 lines. Here, the graph's node set  $V = \{v_1, v_2, \dots, v_N\}$  represents the set of stations, where  $N$  is the  
 24 number of stations. The set of edges  $E$  represents the connections between stations, with an edge  
 25 existing between each pair of adjacent stations.

1           **Definition 2** Adjacency Matrix  $\mathbf{A}^{N \times N}$ . The adjacency matrix  $\mathbf{A} \in \mathbb{R}^{N \times N}$  is a binary matrix  
 2 that represents the connections between stations in the public transport network  $G$ . Each element  
 3  $a_{ij}$  in the matrix is defined as follows:

$$4 \quad a_{ij} = \begin{cases} 1 & \text{if there is an edge between station } v_i \text{ and station } v_j, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

5           **Definition 3** Feature Matrix  $\mathbf{X}^{N \times M}$ . Short-term popularity sequence is considered an at-  
 6 tribute of network nodes and is represented by the feature matrix  $\mathbf{X} \in \mathbb{R}^{N \times S}$ , where  $S$  denotes the  
 7 length of the historical time series. At time  $t$ ,  $X_t \in \mathbb{R}^{i \times N}$  represents the popularity at each node,  
 8 while  $i$  corresponds to the dimensionality of the attribute data, accommodating scenarios where  
 9 there are multiple attributes.

10           **Definition 4** Multi-step popularity forecasting. In the context of multi-step popularity  
 11 forecasting, the goal is to learn a function  $f$  that maps the public transport network  $G$  and the  
 12 feature vector  $\mathbf{x} \in \mathbb{R}^{1 \times N}$  (in some case, feature matrix  $\mathbf{X} \in \mathbb{R}^{i \times N}$  considering the multiple attribute  
 13 features) to future  $T$ -step predictions of popularity for each station. This can be formally described  
 14 as:

$$15 \quad (\mathbf{x}_{t+1}, \dots, \mathbf{x}_{t+T}) = f(G, (\mathbf{x}_{t-s}, \dots, \mathbf{x}_{t-1}, \mathbf{x}_t)) \quad (5)$$

16 where  $s$  denotes the length of the historical popularity sequences, and  $T$  represents the number of  
 17 future steps to be predicted.

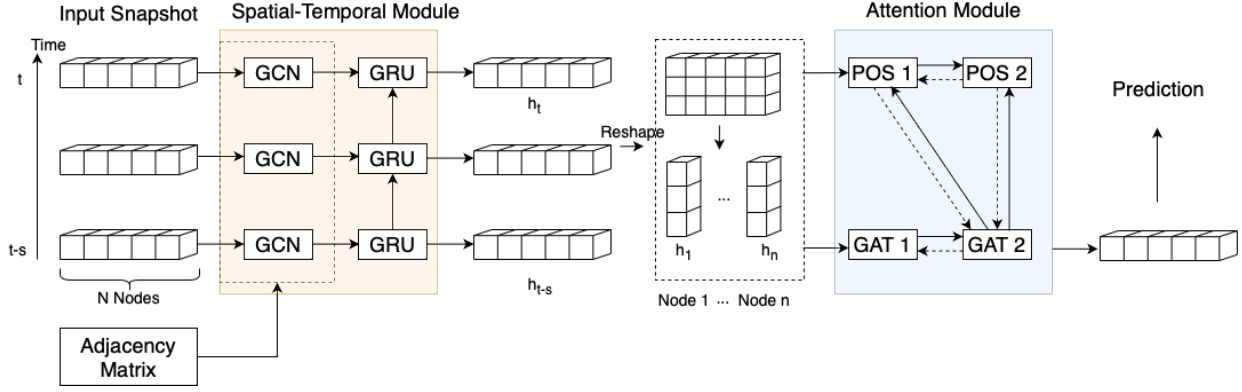
## 18 **Overview of the model architecture**

19 In this study, we proposed a spatial-temporal model with a graph attention module enhanced by  
 20 position embedding mechanism (APT-GCN). Figure 3 illustrate the structure of the APT-GCN  
 21 model. In the first part, the method to capture temporal-spatial dependencies is based on the Tem-  
 22 poral Graph Convolutional Network (TGCN) proposed by (8). It begins by taking time series  
 23 data of popularity, with a length  $s$  for each node, and the adjacency matrix  $\mathbf{A}$  as input. A graph  
 24 convolution network (GCN) is then used to capture spatial features from this data. The output  
 25 of the GCN is subsequently fed into a gated recurrent unit (GRU), which allows for the tempo-  
 26 ral flow of information across different time snapshots. Following the T-GCN module, the output  
 27 is further processed by an attention mechanism designed to integrate historical information from  
 28 non-adjacent stations. This is achieved through a position embedding that enhances the attention  
 29 module's ability to incorporate information from distant nodes. Finally, the results from the atten-  
 30 tion module are aggregated along the temporal dimension to produce forecasts for the next  $T$  time  
 31 steps. This model effectively combines short-term historical data for each node with information  
 32 from other nodes in the graph, enabling more accurate predictions of future popularity trends.

## 33 **Combining spatial-temporal popularity data with special event data**

34 We will add an event label to the popularity data to capture the unique passenger flow character-  
 35 istics during special events. For each special event, the start and end times have been recorded.  
 36 Using this information, we will introduce a new feature, 'event,' to the popularity data. The spe-  
 37 cific indicator is expressed in 6, designating a buffer time of 2 hours before the start and after the  
 38 end of the event. The event feature for the corresponding station  $j$  will be set to 1 if it falls within  
 39 this time range, and 0 otherwise.





**FIGURE 3:** An overview of the attention with positional embedding enhanced temporal convolutional (APT-GCN) network.

$$1 \quad event_{ij} = \begin{cases} 10 - i & \text{if } t_{ij} \in (t_{start} - i \times 30\text{minutes}, t_{end} + i \times 30\text{minutes}) \wedge v_j \in \mathbf{L}_m, i < 10 \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

2 where  $t_{ij}$  is the time stamp  $i$  of station  $j$ ,  $t_{start}$  and  $t_{end}$  represent the start time and end time of event  
3  $m$ ,  $\mathbf{L} + m$  is the public transport line corresponding to event  $m$ .

#### 4 **Gathering neighbours' influence via graph convolution layer**

5 Graph Convolutional Networks (GCNs) are a type of semi-supervised model introduced by Kipf  
6 and Welling in 2016 (9). GCNs enhance traditional CNNs within the domain of graph neural  
7 networks. While CNNs can capture local spatial features in Euclidean space, such as those in  
8 images, they fall short in domains like transportation where spatial dependencies arise from net-  
9 work topological structures. GCNs address this limitation by performing convolution operations  
10 on non-Euclidean data.

11 The fundamental idea of GCNs is to aggregate features from neighboring nodes and then  
12 transform these features. By stacking  $k$  layers, a GCN can capture the features of  $k$ -order neigh-  
13 bors. Given the adjacency matrix  $\mathbf{A}$ , which represents non-Euclidean graph data, a GCN typically  
14 performs two operations: propagation and transformation. Propagation involves using filters in the  
15 Fourier space to capture and aggregate features from first-order neighbors. The aggregated spatial  
16 information is then transformed between layers through linear transformations or activation func-  
17 tions. Given the feature matrix  $\mathbf{X}$  and the matrix  $\tilde{\mathbf{A}}$  representing the network structure, the output  
18 of GCN layer  $l + 1$  is computed as follows:

$$19 \quad H^{(l+1)} = \sigma \left( \tilde{\mathbf{D}}^{-\frac{1}{2}} \tilde{\mathbf{A}} \tilde{\mathbf{D}}^{-\frac{1}{2}} H^{(l)} \theta^{(l)} \right) \quad (7)$$

20 where  $\tilde{H}^{(l)}$  is the output of layer  $l$ ,  $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{I}$  denotes adjacency matrix adding self-connection,  
21  $\mathbf{I} \in \mathbb{R}^{N \times N}$  is an identity matrix.  $\tilde{\mathbf{D}}$  is the degree matrix computed by  $\tilde{\mathbf{D}} = \sum_j \tilde{\mathbf{A}}_{ij}$ .  $\theta^{(l)}$  denote the  
22 parameter of layer  $(l + 1)$ .  $\sigma(\cdot)$  represents the activation function.

23 Thus, a graph convolutional network can be defined as:

$$24 \quad H^{(l)} = f(H, \hat{\mathbf{A}}) = \sigma \left( \hat{\mathbf{A}} H^{(l-1)} W^{(l)} \right) \quad (8)$$

$$Z = f^{(n)}(X, \hat{A}) = \underbrace{f(f(\dots f(X, \hat{A})\dots))}_{n \text{ layers}} \quad (9)$$

$\hat{A} = \tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}}$  which is a pre-process of adjacency matrix.  $\mathbf{W}^{(l)} \in \mathbb{R}^{S \times C}$  denoted the weight of layer  $l$ , where  $S$  is the length of historical popularity series, and  $C$  denotes number of units.  $n$  is the number of layer, to capture spatial dependencies from  $n$ -order neighbours. In our model,  $n$  is set to be 2.  $\mathbf{X}$  is input feature matrix and  $\mathbf{Z} \in \mathbb{R}^{N \times C}$  is the final output of GCN network and  $C$  is the number of units from output layer.

### 8 Gated recurrent units for temporal domain

When addressing transportation issues—whether they pertain to road traffic, rail transit, or a combination of public transportation types—participants invariably move through a specific network along the time dimension. In the short term, historical data significantly influences the prediction of traffic related variables at a given moment. This relationship has been highlighted in various studies, which emphasizes the importance of incorporating historical data to enhance the accuracy of traffic forecasts (10). Therefore, one of the primary objectives in traffic forecasting is to capture temporal dependencies effectively. This goal can be achieved using RNNs, which are designed to handle sequence data and capture dependencies across time. RNNs have been extensively studied for their ability to model temporal sequences and their applications in time-series forecasting.

RNNs come in several variants, with Long Short-Term Memory (LSTM) units and Gated Recurrent Units (GRUs) being among the most notable. LSTM units were introduced by Hochreiter and Schmidhuber in 1997(11) to address the vanishing gradient problem in traditional RNNs (12). LSTMs use a more complex structure involving input, forget, and output gates to control the flow of information, which helps them maintain long-term dependencies. On the other hand, GRUs, introduced by Cho et al. in 2014, simplify this structure by combining the input and forget gates into a single update gate, making them computationally less demanding (13). While both models utilize gating mechanisms to control the flow of information, LSTMs generally involve more complex operations, which can lead to longer training times (14). In our study, we evaluated the performance of GRUs and LSTMs on sample data, as well as the combination of these units with single-layer models. Experimental results indicated that while LSTM and the combined model did not significantly improve accuracy under the same conditions, they considerably increased the training time. During our experiment on several lines, the GRU layer achieved an accuracy of 0.9145, while the LSTM layer option slightly improved accuracy to 0.9181, marking a 0.39% increase. However, this improvement in accuracy came at the cost of a 19.16% increase in training time, rising from 2696.31 seconds to 3213.82 seconds. Therefore, considering the trade-offs between accuracy and computational efficiency, we ultimately chose GRUs as the recurrent units to capture temporal information in our model. The GRU cell processes the output  $\mathbf{Q} \in \mathbb{R}^{N \times C}$  from graph convolutional network by:

$$Q_t = (1 - U_t) Q_{t-1} + U_t C_t \quad (10)$$

$U_t$  is the update gate to control information from last time step, it along with memory content at current moment  $C_t$  are computed as:

$$1 \quad U_t = \sigma(W_U[Z_t, Q_{t-1}] + b_U) \quad (11)$$

$$2 \quad R_t = \sigma(W_R[Z_t, Q_{t-1}] + b_R) \quad (12)$$

$$3 \quad C_t = \tanh(W_C Z_t + W_C(R_t Z_{t-1}) + b_C) \quad (13)$$

4 where  $\mathbf{Z}_s \in \mathbb{R}^{N \times C}$ ,  $s \in (0, S)$  is the output of last graph convolutional section. Comparing the  
5 original GRU unit, here adding  $\mathbf{R}_t \in \mathbb{R}^{N \times C}$  as the reset gate, considering the degree of ignoring  
6 information from early time steps.

7 The output of the GRU, incorporating temporal dynamics, will be passed to the subsequent  
8 attention module. Noticeably, the output  $\mathbf{Q}$  from the previous module has not aggregated the  $s$  time  
9 steps along the time sequence, thus still retaining the information in the time series dimension.

### 10 Graph attention module for enhancing non-Local dependencies

11 Crowdedness forecasting in public transport systems, as a subset of general traffic prediction, in-  
12 herits characteristics from traffic forecasting while presenting unique challenges. Public transport  
13 operates on a fixed service schedule, leading to discrete passenger counts at each station and dis-  
14 tinctive passenger flow dynamics. This discrete nature becomes particularly pronounced during  
15 special events, where sudden surges in passenger volumes can influence multiple subsequent time  
16 steps and nodes within the network (15).

17 In previous modules, we utilized graph convolutional networks (GCNs) to capture spa-  
18 tial features within the network. However, a single-layer GCN aggregates information only from  
19 immediate neighbors, neglecting the flow of popularity between non-adjacent stations. This limita-  
20 tion is particularly critical in scenarios where rail public traffic passenger transfers between distant  
21 stations occur rapidly. For instance, the total trip duration on Munich’s U-Bahn Line 6, which  
22 comprises 27 stations, is approximately 51 minutes(16). Even considering the normal operation  
23 schedule is every 10 minutes on this line, this relatively short duration means that passengers can  
24 traverse almost the entire line within a single data collection interval, posing a challenge for single  
25 or dual layer GCN to capture neighbour node’s features. To address this, merely stacking GCN  
26 layers to capture higher-order neighbor information is inefficient and computationally expensive,  
27 given that the stations of urban public lines are usually beyond 20-30. Moreover, GCNs lack the ca-  
28 pability to assign different weights to different nodes, limiting their effectiveness in heterogeneous  
29 networks.

30 To overcome these limitations, we introduce a graph attention module to capture non-local  
31 dependencies effectively. The graph attention mechanism allows the model to weight the impor-  
32 tance of each node differently, thus providing a more nuanced representation of the network. This  
33 module is further enhanced with a position embedding mechanism, which helps in integrating  
34 historical information from non-topologically adjacent stations into the current node’s prediction.  
35 This mechanism is particularly effective for non-homophilic graphs(17), enabling us to capture  
36 features from stations that are not geographically or topologically proximate but share similar  
37 characteristics. This significantly enhances the accuracy of popularity forecasting, especially un-  
38 der special event conditions. During such events, passenger movement along public transport lines  
39 tends to follow more predictable patterns, often involving transfers from major interchange sta-  
40 tions, such as airports or train stations, to event-specific locations. This consideration is crucial for  
41 improving the precision of our forecasts.

42 The integration of a graph attention module thus not only improves the capture of spatial

1 dependencies across distant nodes but also ensures that the model remains efficient and effective.  
 2 This approach is particularly vital for scenarios involving special events, where unexpected spikes  
 3 in passenger demand necessitate a robust model capable of dynamic and rapid adaptation. This  
 4 module is based on the Graph Attentional Networks with Positional Embeddings (GAT-POS) pro-  
 5 posed by Ma et al. (17). It comprises two graph attention layers and two positional embedding  
 6 layers that interact with each other, enhancing the model’s ability to capture complex spatial rela-  
 7 tionships within the data. Taking the output  $\mathbf{Q}$  from last section and reshaping it into vectors of  
 8 node features  $\mathbf{q}_v \in \mathbb{R}^{S \times C}$  as input, and  $\mathbf{q}'_v \in \mathbb{R}^{1 \times C}$  as expected output, the attention coefficients is  
 9 computing by

$$10 \quad e_{vu}^k = a(W_k q_v + U_k p_v \parallel W_k h_u + U_k p_u) \quad (14)$$

11 where  $\mathbf{W}_k$ ,  $\mathbf{U}_k$  and  $\mathbf{a}_k$  are the weights in the  $k$ -th attention head.  $\parallel$  is the concatenation operation.  
 12  $\mathbf{p}_v$  is the positional embedding for node  $v$  computed by,

$$13 \quad p_v^l = \sigma(W_{emb}^l p_v^{l-1}) \quad (15)$$

$$14 \quad \mathbf{L}(p_{v \in \mathcal{N}}, G) = \sum_{v \in \mathcal{N}} \sum_{u \in \mathcal{N}(v)} (-\log \sigma(p_v^T p_u) - Q \cdot \mathbb{E}_{u' \in P_n(v)} \log(\sigma(-p_v^T p_{u'}))) \quad (16)$$

15 where  $l$  is the  $l$ -th position embeddings layer, and  $\mathbf{W}_{emb}^l$  is the learned weight matrix of position  
 16 embeddings layer  $l$ . Equation 16 describes loss function of the unsupervised embedding model.

17 The attention coefficients  $\mathbf{e}_{vu}$  are normalized by softmax function:

$$18 \quad \alpha_{vu}^k = softmax_u(e_{vu}) = \frac{\exp(e_{vu})}{\sum_{j \in \mathcal{N}_v} \exp(e_{vj})} \quad (17)$$

19 Then, a linear transform is applied to gain the output  $\mathbf{q}'_v$  by combining neighbours with  
 20 normalized coefficients,

$$21 \quad \mathbf{q}'_v = \sigma\left(\frac{1}{K} \sum_{k=1}^K \sum_{u \in \mathcal{N}_v} \alpha_{vu}^k W^k \mathbf{q}_u\right) \quad (18)$$

22 Finally, a fully connection layer is used to combined the  $S$  time step and make prediction  
 23 on future  $T$ -step. In conclusion, the APT-GCN model effectively captures the spatial-temporal  
 24 dependencies in public traffic networks and efficiently predicts future popularity. The first module  
 25 captures the spatial information of topologically adjacent nodes on the public transit lines, as well  
 26 as the historical information of the time series. The second module utilizes a graph attention net-  
 27 work with position embedding mechanisms to obtain information on semantically adjacent stations  
 28 in crowdedness patterns.

## 30 RESULTS

31 In this section, we employ the collected and processed GPT datasets to analyze the popularity of  
 32 various urban public transit stations. Using this data, we train the APT-GCN model on each public  
 33 transport network and present the forecasting results.

### 34 Setting Model Parameters

35 The model is implemented using TensorFlow 2.16.1, running on a Mac Book Pro with an Apple  
 36 M2 Max chip with 32 GB of memory and 12 cores. The training process encompasses 1000  
 37 epochs, utilizing a batch size of 128. The dataset is partitioned into training, validation, and test  
 38 sets, adhering to a proportion of 7:1:2. The optimization of the model is facilitated by the adaptive  
 39 moment estimation (Adam) optimizer, with a learning rate set at 0.001. Furthermore, the units of

1 the hidden layers are configured to 64. The historical time series data length  $S$  is 8, and the model  
 2 performs one-step predictions with the prediction length  $T$  set to 1.

### 3 **Evaluation Metrics**

4 The following error metrics are introduced to evaluate performance on popularity prediction of the  
 5 APT-GCN model:

- 6 • Root Mean Squared Error (RMSE):

$$7 \quad \text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^M (\mathbf{y}_i - \hat{\mathbf{y}}_i)^2} \quad (19)$$

- 8 • Mean Absolute Error (MAE):

$$9 \quad \text{MAE} = \frac{1}{M} \sum_{i=1}^M |\mathbf{y}_i - \hat{\mathbf{y}}_i| \quad (20)$$

- 10 • Coefficient of Determination ( $R^2$ ):

$$11 \quad R^2 = 1 - \frac{\sum_{i=1}^M (\mathbf{y}_i - \hat{\mathbf{y}}_i)^2}{\sum_{i=1}^M (\mathbf{y}_i - \bar{\mathbf{y}})^2} \quad (21)$$

- 12 • Explained Variance Score (Var):

$$13 \quad \text{Var} = 1 - \frac{\text{Var}(\mathbf{Y} - \hat{\mathbf{Y}})}{\text{Var}(\mathbf{Y})} \quad (22)$$

- 14 • Accuracy:

$$15 \quad \text{Accuracy} = 1 - \frac{\|\mathbf{Y} - \hat{\mathbf{Y}}\|_F}{\|\mathbf{Y}\|_F} \quad (23)$$

16 where  $M$  is the number of samples,  $\mathbf{y}_i$ ,  $\hat{\mathbf{y}}_i$  and  $\bar{\mathbf{y}}_i$  represent the real, prediction and average popularity  
 17 vector of network of  $i$ th sample.  $\mathbf{Y}$  and  $\hat{\mathbf{Y}}$  is the set of  $\mathbf{y}_i$  and  $\hat{\mathbf{y}}_i$ .  $\|\cdot\|$  represents the Frobenius norm.

## 18 **Experiment Results**

### 19 *Overview*

20 Table 4 presents the prediction results of the model across the entire public transport network. For  
 21 each line, the final results of the evaluation metrics are derived from the arithmetic mean of all  
 22 the stations on that line. The table provides the evaluation results for both regular conditions and  
 23 special event scenarios. Since multiple events occurred during the data coverage period for each  
 24 line, the event results presented in this table are also arithmetic means of the results from all events.

**TABLE 4:** The Evaluation of Crowdedness Forecasting on GPT dataset under Regular and Special Event Scenario

Evaluation Matrix	Scenario	rmse	mae	r2	var	acc
Line 1	Regular	0.0299	0.0226	0.9511	0.9525	0.9132
	Special Event	0.0243	0.0182	0.9554	0.9586	0.9215
Line 7	Regular	0.0484	0.0356	0.9183	0.9184	0.8804
	Special Event	0.0462	0.0351	0.8952	0.9045	0.8843
Line 10	Regular	0.026	0.018	0.924	0.9244	0.8806
	Special Event	0.0254	0.017	0.898	0.9026	0.8546
Line 12	Regular	0.0288	0.0217	0.9603	0.9603	0.9157
	Special Event	0.0254	0.0198	0.9491	0.9545	0.906
Blue	Regular	0.0526	0.0406	0.9109	0.9138	0.861
	Special Event	0.0503	0.0378	0.895	0.9097	0.859
District	Regular	0.0391	0.0301	0.8978	0.898	0.8834
	Special Event	0.0303	0.0232	0.8744	0.8829	0.8989
Piccadilly	Regular	0.0492	0.0383	0.8662	0.8668	0.8707
	Special Event	0.0415	0.0318	0.8734	0.8745	0.8794
M2	Regular	0.0235	0.0151	0.983	0.9831	0.9099
	Special Event	0.0401	0.0236	0.9662	0.9673	0.8744
M3	Regular	0.0416	0.0312	0.915	0.9155	0.889
	Special Event	0.0473	0.0351	0.8677	0.8737	0.8713
S3	Regular	0.0224	0.017	0.9011	0.9019	0.8652
	Special Event	0.0188	0.014	0.9006	0.9143	0.8775
Tram1	Regular	0.016	0.012	0.9579	0.9585	0.919
	Special Event	0.0211	0.0159	0.872	0.8761	0.9061
Tram3	Regular	0.051	0.0376	0.9525	0.9528	0.8693
	Special Event 2	-	-	-	-	-
U6	Regular	0.0465	0.0346	0.8764	0.8765	0.8564
	Special Event	0.032	0.0252	0.8833	0.8975	0.9009
U45	Regular	0.0471	0.0369	0.8148	0.8169	0.8253
	Special Event 2	-	-	-	-	-
U46	Regular	0.0406	0.0277	0.9533	0.9539	0.8724
	Special Event 2	-	-	-	-	-
Yellow	Regular	0.026	0.0195	0.9569	0.9571	0.9094
	Special Event	0.0249	0.0185	0.9408	0.9423	0.9199

1 For the transport lines Tram 3 in Augsburg, Germany, and U45 and U46 in Dortmund,  
 2 Germany, no valid data under special events was available during our collection period; therefore,  
 3 the results for special events are missing for these lines. Despite this, examining the prediction  
 4 performance for other lines under special event conditions reveals a notable improvement.

#### 5 *Robustness during special events*

6 In our experiments, we initially tested two models. One considers only the popularity feature  
 7 for each time series and another additionally includes the encoded event feature. The results for  
 8 Madrid metro Line 1, Line 7 and Munich metro U6 present in Table 5, indicating that while the  
 9 inclusion of the event feature might lead to a decrease in overall performance across the entire test  
 10 set, such as with Line 1, Line 7 and U6 where the average accuracy of the second model decreased  
 11 by 0.94%, 1.03% and 0.94% respectively, it significantly improves performance in special event  
 12 scenarios by 216.78%, 20.04% and 54.21%. This demonstrates that our model is more robust  
 13 in capturing crowdedness patterns during special events, enhancing the accuracy of popularity  
 14 predictions under these conditions. This improvement is visualized in Figure 4, which clearly  
 15 shows the enhanced prediction accuracy of our model during special events.

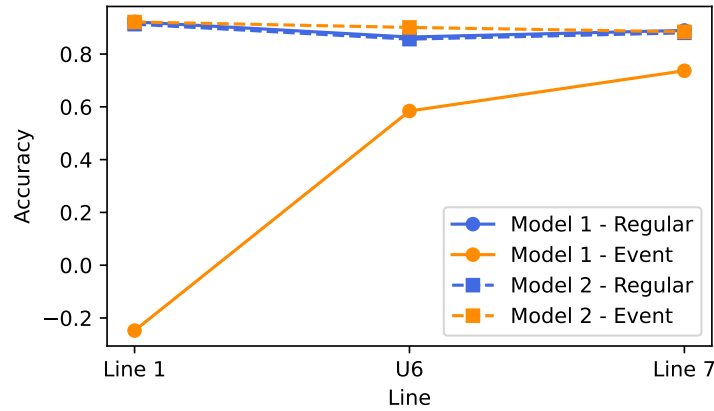
**TABLE 5:** Performance Comparison of Different Models

Line	Scenario	Model	rmse	mae	r2	var	acc
Line 1	Regular	Model 1	0.027	0.0201	0.9603	0.9603	0.9218
		Model 2	0.0299	0.0226	0.9511	0.9525	0.9132
	Special Event	Model 1	0.6417	0.6397	-67.1586	0.5764	0.2909
		Model 2	0.0243	0.0182	0.9554	0.9586	0.9215
U6	Regular	Model 1	0.0439	0.0338	0.89	0.8901	0.8645
		Model 2	0.0465	0.0346	0.8764	0.8765	0.8564
	Special Event	Model 1	0.1927	0.151	-0.0924	-0.0841	0.5842
		Model 2	0.032	0.0252	0.8833	0.8975	0.9009
Line 7	Regular	Model 1	0.0447	0.0331	0.9303	0.931	0.8895
		Model 2	0.0484	0.0356	0.9183	0.9184	0.8804
	Special Event	Model 1	0.1647	0.1235	0.2617	0.3587	0.7367
		Model 2	0.0462	0.0351	0.8952	0.9045	0.8843

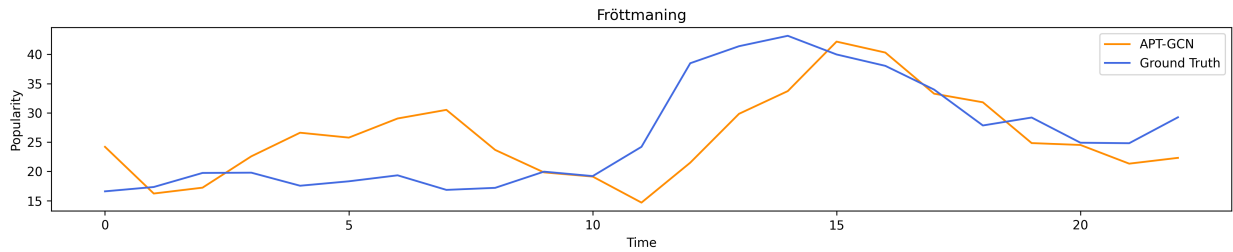
#### 16 *Interpretation and visualisation of crowdedness pattern*

17 To further examine the details of the experiment results, Figure 5a visualizes the prediction re-  
 18 sults for a Germany Bundesliga (first-class) football match on a weekday. Specifically, Figure 5b  
 19 shows a comparison between the predicted results(?) and the ground truth on the training set is  
 20 demonstrating.

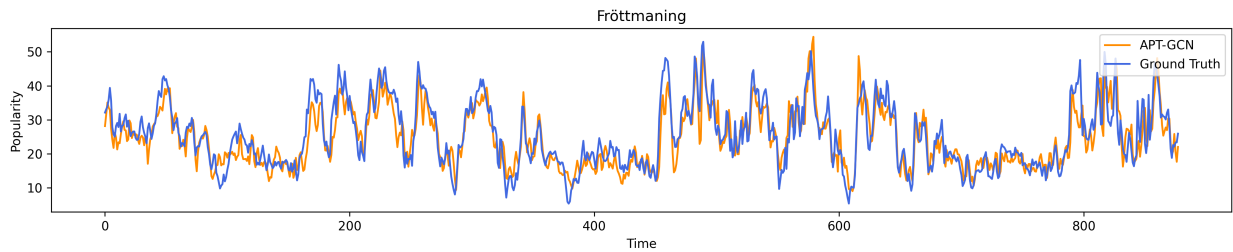
21 This result represents a typical crowdedness pattern at public transport stations near venues  
 22 during special events. Popularity increases and peaks approximately two hours before the event  
 23 begins (as observed in Munich), decreases during the event, and then reaches another peak im-



**FIGURE 4:** Model Performance Analysis



(a) Prediction During Special Event



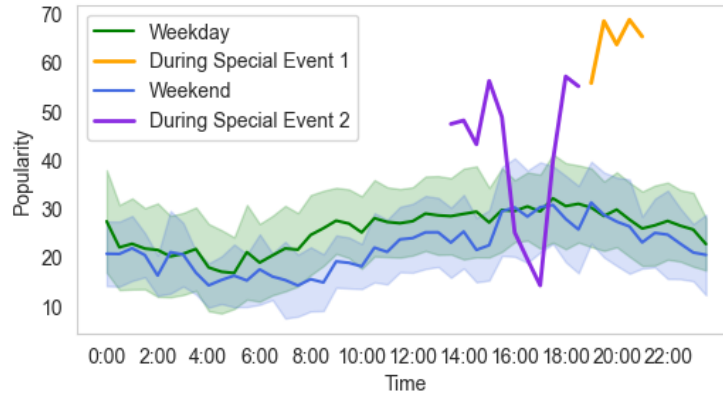
(b) Prediction on Training Set

**FIGURE 5:** Forecasting Crowdedness at Munich Metro Line U6 station Fröttmaning

1 mediately after the event ends as passengers gradually disperse over time. Figure 6 illustrates the  
 2 popularity comparison between special event periods and average value on weekday and weekend,  
 3 with Special Event 1 representing an event held on a weekend and Special Event 2 representing an  
 4 event held on a weekday.

5 When examining the entire public transport line, it is evident that crowdedness patterns  
 6 vary significantly across different events, cities, and networks. Figure 7(a) illustrates the distribu-  
 7 tion of popularity on the Madrid Metro Line 7 during the La Liga match between Atletico Madrid  
 8 and Betis on March 3, 2024. This match began at 16:15 and ended at 18:15. The blue font corre-  
 9 sponds to the station Estadio Metropolitano, which is closest to the stadium. Surrounding stations  
 10 experienced an increase in popularity before and after the match, whereas Estadio Metropolitano  
 11 itself did not show a significant increase, potentially due to the station’s design.



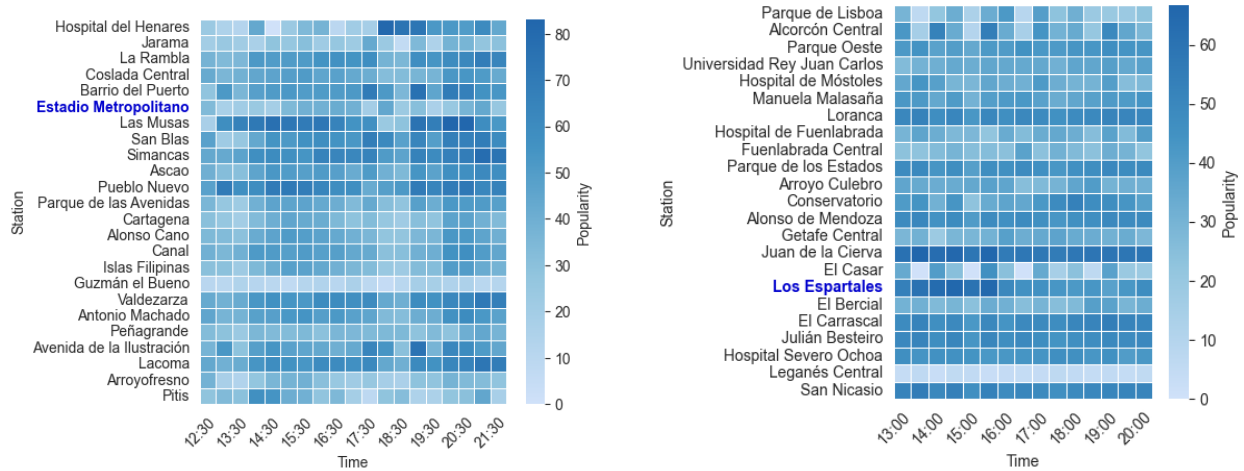


**FIGURE 6:** Popularity Comparison during Special Event and Regular Times at Munich Metro Line U6 station Fröttmaning

1 Figure 7(b) represents the match between Getafe and Mallorca on May 26, 2024, on the  
 2 Madrid Metro Line 12. Los Espartales, the station near the stadium, exhibited crowding during the  
 3 event. The crowding effect extended to specific upstream and downstream stations on the line.  
 4 Figure 7(c) presents data from the Premier League match between Newcastle and Wolves  
 5 on March 2, 2024. The corresponding line is the Newcastle Metro Yellow Line. According to the  
 6 Newcastle subway network map (Figure 8), St James, the station near the stadium, is located at  
 7 one end of this circular line. Other stations on the line showed passenger flow patterns typical of  
 8 regular times, indicating limited impact from the event.  
 9 Figure 7(d) highlights the match between Arsenal and Chelsea on April 23, 2024, on the  
 10 London Tube Piccadilly Line. The special event significantly influenced popularity at Arsenal and  
 11 nearby stations, displaying a strong spatiotemporal correlation with the event’s timing.

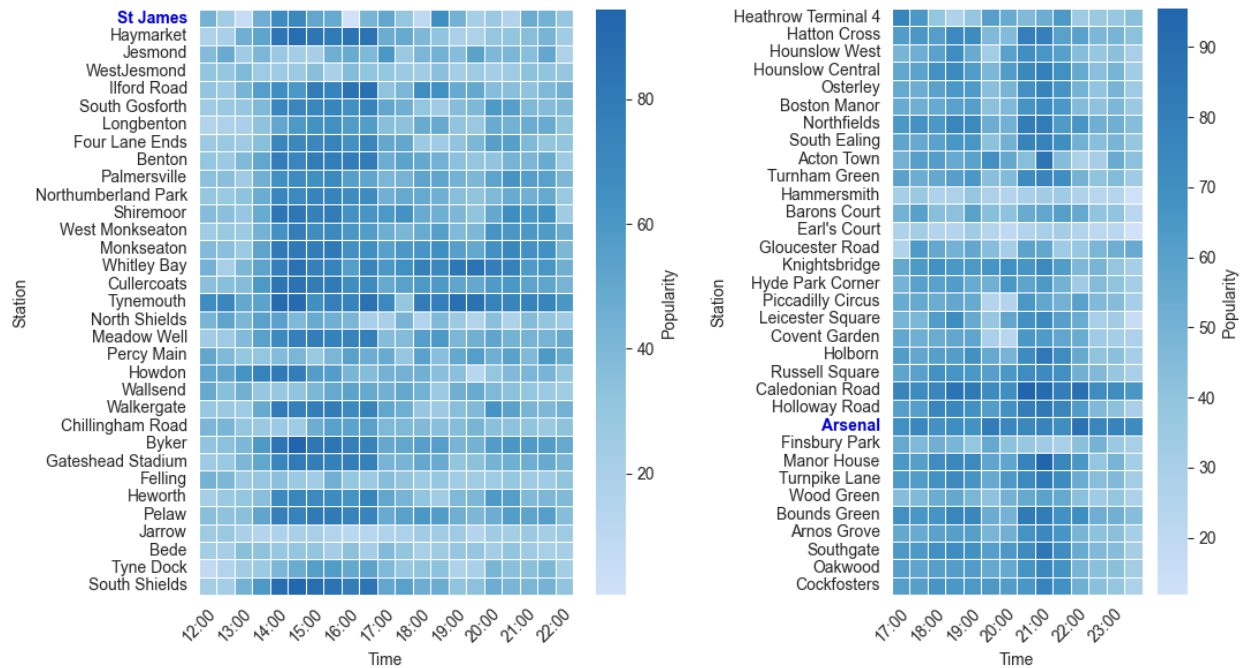
## 12 CONCLUSIONS

13 Our study analyzes the crowdedness patterns at public transport stations during special events  
 14 across multiple cities and public transport networks by constructing a spatial-temporal graph neu-  
 15 ral network model. Recognizing that sensor data (e.g., IC card data) is not always available or  
 16 open-source and digging into the potential of opportunistic data, we have constructed a dataset  
 17 comprising 428 public transport stations across 17 lines in 10 cities derived from Google Popu-  
 18 lar Time data. Our model is designed to handle the abnormal passenger flow that special events  
 19 bring, a significant departure from regular traffic patterns. This unique feature of our model as-  
 20 sures its robustness in the face of short-term forecasting challenges. We’ve also considered the  
 21 inherent spatial characteristics of the traffic network, integrating the topological structure of rail  
 22 lines and the adjacency relationships between stations into the temporal dependencies. In our  
 23 experiments, we initially compared models that considered the event feature with those that did  
 24 not. We demonstrated that the model incorporating the designated special feature significantly  
 25 improved performance under special event scenarios without substantially compromising overall  
 26 accuracy. We showcased the model’s predictive performance across all networks and visualized  
 27 the results, along with the visualization and analysis of popularity change under different scenar-  
 28 ios. Our research underscores the importance of including special event data in passenger flow  
 29 prediction. The APT-GCN model effectively addresses the challenges posed by special events,



(a) During Special Event on Line 7

(b) During Special Event on Line 12



(c) During Special Event on Yellow Line

(d) During Special Event on Piccadilly Line

**FIGURE 7: Crowdedness Pattern during Special Events**

1 providing valuable insights for urban rail transit management. Future work can explore additional  
 2 data sources and refine the model to enhance prediction accuracy and operational efficiency across  
 3 urban environments.

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 6 of Technical University of Munich (TUM) (MODA project).

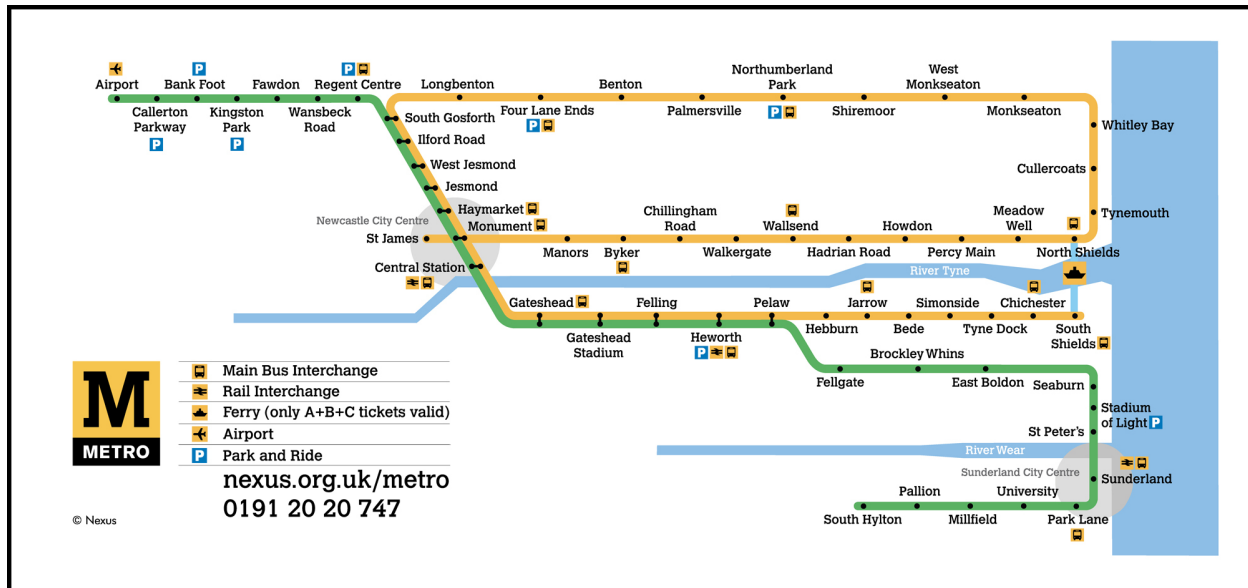


FIGURE 8: Newcastle Public Railway Transport Network Map

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